AN

ELEMENTARY TREATISE

ON

SOUND;

BEING THE SECOND VOLUME

OF A

COURSE OF NATURAL PHILOSOPHY,

DESIGNED FOR

THE USE OF HIGH SCHOOLS AND COLLEGES.

COMPILED BY

BENJAMIN PEIRCE, A. M.,

UNIVERSITY PROFESSOR OF MATHEMATICS AND NATURAL PHILOSOPHY IN HARVARD UNIVERSITY.

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ADVERTISEMENT.

THE present work lays no claim to originality. It is, essentially, the able and finished treatise, written by Sir John Herschel for the Encyclopedia Metropolitana, adapted to the purposes of instruction, with such changes as are demanded by some later investigations. Thus the analysis of the Vibrations of Plates, in Part II., Chapter V., is taken from Wheatstone's Memoir in the London Philosophical Transactions for 1833; the whole of Part III., Chapter III., is from Faraday's Memoir in the Transactions for 1831; and Chapters IV. and V. of Part III. are almost entirely rewritten from other sources, such as Milne-Edwards's Elements of Zoology; Breschet's Anatomy and Physiology of the Ear, in the Annales des Sciences Naturelles of Paris for 1833; Sir Charles Bell on the Organs of the Human Voice, in the London Philosophical Transactions for 1832; Willis on the Mechanism of the Larynx, in the Cambridge Philosophical Transactions for 1833; Bennati's Mechanism of the Human Voice; and Rush's Treatise on the Human Voice; combined with some original remarks.

The catalogue of writings on Sound in the Table of Contents is prepared with the greatest care from the Catalogue of Dr. Young, in his Lectures on Natural Philosophy, and from an examination of every volume of the Memoirs of the Academies of Paris, Turin, Berlin, Petersburgh, Göttingen, Philadelphia, Boston, of the Philosophical Transactions of London, Edinburgh, Dublin, Cambridge, the Journal de Physique by M. Rosier, the Journal Polytechnique, Annales des Mathematiques, various Catalogues and Journals, and the works of Bernoulli, D'Alembert, Cauchy, &c.

BENJAMIN PEIRCE.

Cambridge, 1836.

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INSTRUMENTS FOR HEARING.

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SOUND.

PART I.

ON THE PROPAGATION OF SOUND IN GENERAL.

CHAPTER I.

ON THE PROPAGATION AND VELOCITY OF SOUND IN AIR.

1. To explain the nature and production of sound, the laws of its propagation through the various media which convey it to our ears, and the manner of its action on those organs; the modifications of which it is susceptible in speech, in music, or in inarticulate and unmeaning noises; and the means, natural or artificial, of producing, regulating, or estimating them, are the proper objects of *Acoustics.*

2. Sound will not pass through a void space, but is almost always conveyed to the ear by means of the air; and it diminishes in intensity by passing through rarefied air, while it becomes more intense by passing through condensed air.

Shortly after the invention of the air-pump it was found, that the collision of hard bodies in an exhausted receiver produced no appreciable sound. Hauksbee, having suspended a bell in the

receiver of an air-pump, found the sound die away by degrees, as the air was exhausted, and again increase on its readmission; and when made to sound in a vessel full of air, the sound was not transmitted through the interval between that and an exterior vessel from which the air had been extracted, though it passed freely when the air was readmitted. On the other hand, when the air was condensed in a receiver, the sound of a suspended bell was stronger than in natural air, and its intensity increased with the degree of condensation. Roebuck, when shut up in a cavity excavated in a rock, which served as a reservoir of air for an iron foundry in Devonshire to equalize the blast of the bellows, observed the intensity of sound to be considerably augmented in the air thus compressed by their action. The same effect has been experienced in diving bells. More recently, M. Biot has repeated the experiment of the exhausted receiver, with a much more perfect vacuum than could be procured in Hauksbee's time, and found the sound to be quite imperceptible, even when the ear was held close to the receiver, and all attention paid.

3. Sound is less intense at great elevations in the atmosphere.

The diminution of the intensity of sound in a rarefied atmosphere is a familiar phenomenon to those who are accustomed to ascend very high mountains. The deep silence of those elevated regions has a physical cause, independent of their habitual solitude. Saussure relates, that a pistol, fired on the summit of Mont Blanc, produced no greater report than a little Indian cracker would have done in a room. Herschel noticed the comparatively small extent to which the voice can be heard, at an altitude of upwards of 13,000 feet on Monte Rosa. The height, however, to which an atmosphere, or medium capable of conveying sound, extends, far exceeds any attainable on mountains, by balloons, or even by the lightest clouds. The great meteor of 1783 produced a distinct rumbling sound, although its height above the earth's surface was full 50 miles at the time of its explosion. The sound produced by the explosion of the meteor of 1719, at an elevation of at least

69 miles, was heard as "the report of a very great cannon or broadside," shook the windows and doors of houses, and threw a looking-glass out of its frame and broke it. These heights are deduced by calculation from observations too unequivocal, and agreeing too well with each other, to allow of doubt. Scarcely less violent was the sound caused by the bursting of the meteor of July 17, 1771, near Paris; the height of which, at the moment of the explosion, is assigned by Le Roy at about 25 miles. The report of a meteor, in 1756, threw down several chimneys at Aix in Provence, and was taken for an earthquake. These instances, and others which might be adduced, are sufficient to show that sound can be excited in, and conveyed by, air of an almost inconceivable tenuity, (for such it must be at the heights here spoken of,) provided the exciting cause be sufficiently powerful and extensive; neither of which qualities can be regarded as deficient in the case of fire-balls, such as those of 1719 and 1783, the latter of which was half a mile in diameter, and moved at the rate of 20 miles in a second. It may, however, be contended, and not without some probability, that at these enormous heights sound may owe its propagation to some other medium more rare and elastic than air, and extending beyond the limits of the atmosphere of air and vapor.

4. Sound is not conveyed instantaneously from the sounding body to the ear. It requires time for its propagation.

This is a matter of the most ordinary observation. We hear the blows of a hammer, at a distance, a very sensible interval of time after we see them struck. The report of a gun is always heard later than the flash is seen, and the interval is longer the more distant the gun. We estimate the distance of a thunder-storm by the length of the interval between the lightning and the thunder-clap, which often arrives when we have ceased to expect it. The report of the meteor of 1783 was heard at Windsor Castle, ten minutes after its disappearance. This is, probably, the longest interval yet observed.

5. The velocity of sound, as determined by the most accurate experiments, is 1090 English feet per second, at the temperature of freezing, and this result does not probably differ more than a foot from the truth. It is, indeed, affected more or less by the state of the air through which the sound passes; but the following are the only circumstances which have any important influence.

I. The direction and velocity of the wind. The velocity of the wind, estimated in the direction of the sound, is to be added to the observed velocity of the sound, to obtain its true velocity, if the wind moves in the opposite direction to the sound; and the velocity of the wind is to be subtracted, if it moves in the same direction with the sound.

II. The temperature of the air through which the sound is conveyed. Every additional degree of atmospheric temperature, on Fahrenheit's scale, adds 1.14 foot to the velocity of the sound; so that at 62° Fahrenheit, which is the standard temperature of the British metrical system, the sound runs over 9000 feet in eight seconds, 12¾ British standard miles in a minute, or 765 miles in an hour, which is about three-fourths of the diurnal velocity of the earth's equator.

a. A great multitude of experiments have been made to determine the precise velocity of sound. The earlier results differ more than might have been expected, from the influence of several causes not immediately obvious, but chiefly from want of due attention to the influence of the wind. It is evident, from the mechanical concussion attending loud noises, that sound consists in a motion of the air itself communicated along it by virtue of its elasticity, as a tremor runs along a stretched rope. If, then, the

whole body of the air were moving in a contrary direction, with the velocity of sound, it would never make its way against the stream at all; and, on the other hand, when the wind blows from the sounding body direct towards the ear, it is equally clear that the velocity of the wind itself will be added to that of sound in still air. If a stone be thrown into a still lake, the waves spread with equal rapidity in all directions, in circles whose centre is the stone. If into a running river, they still form circles, but their centre is carried down the stream; and in point of fact, the wave arrives opposite to a point of the bank above the place where the stone fell, later than at a point at the same distance below it, in proportion to the rapidity of the stream. Hence all experiments on the velocity of sound ought to be made, if possible, either in calm weather, or in a direction at right angles to that of the wind.

b. The first experiments, which appear to have been made with any degree of care, were those instituted by the Florentine Academy Del Cimento. It was observed in these that at a distance equal to 5739 English feet, the sound of a harquebuss arrived five seconds after the flash; and repeating the experiment at half the distance, they found exactly half the time to be required. This gives, for the velocity of sound, 1148 feet per second.

Cassini the Elder, Picard, and Roëmer, from experiments made at a distance of one eighth of a mile, assign 1172; while Flamstead and Halley, from a series of observations at the Royal Observatory, the origin of the sound being three miles distant, concluded the velocity to be 1142 feet per second. Dr. Derham obtained the same result from a more thorough investigation of the whole subject than had ever before been made; and as the distances of the stations employed were considerable, in one case amounting to upwards of 12 miles, this determination appears deserving of some reliance. The temperature unfortunately was not registered; so that the experiment loses much of its value from the impossibility of applying with certainty the requisite correction.

c. In 1737–38, the Academy of Paris directed a re-investigation of the subject, and Messrs. Cassini de Thury, Maraldi, and La Caille, who were at that time engaged in the triangulation of

France, were charged with the conduct of the experiments. Their observations were carefully made, and the distance of the stations was considerable, from 3 to 20 miles. In these experiments we find the first example of observations so disposed, as to eliminate in some measure the disturbing effect of the wind. To apprehend how this may be done, let us suppose a current of wind to blow *uniformly* with any velocity from one station *A* to another *B*, at any distance, and at these two stations let shots be fired. The sound of the shot fired at *A* will then be accelerated, and that of the signal at *B* will be retarded, in traversing the interval, by equal quantities; and consequently (since the velocity of sound is very much greater than that of the most violent wind) the time in which the sound runs over the line from *A* to *B* will be diminished, and that in which it passes from *B* to *A* increased, by nearly equal quantities; so that the mean will be unaffected by the wind's velocity.

d. A great number of exact experiments have been made during the present century by eminent philosophers, and every precaution has been taken to avoid errors, particularly by the Dutch and Parisian observers. The following is a tabular view of their results, reduced to the freezing temperature.

TABLE. Velocity of Sound as determined by various experiments during the present century, and reduced to the freezing temperature.

Observers' Names.	Date of Determination. A. D.	Distance of Stations in Feet.	Velocity in English Feet per second.
Benzenberg (Dusseldorff), . . .	1809	29764	1093
Arago, Matthieu, Prony, Bouvard, Humboldt, ...	1822	61064	1086.1
Moll, Vanbeek, Kuytenbrouwer, .	1823	57839	1089.42
Gregory, ...	1823	various 2700 to 13460	1088.05
Myrbach, ...	1822	32615	1092.1
Goldingham (Madras), ...	1821	29547	1089.9
		13932	1079.9
		Mean	1086.7

The agreement between the above results is very close; their extreme discrepancy being less than 7 feet, or a 160th of the whole amount, and their mean (1089.7) agreeing almost precisely with the result of Moll, Vanbeek, &c.; 1090 feet may therefore be adopted without hesitation, as a whole number which does not differ more from the truth, than was stated at the head of this section. The grounds of the correction for temperature will be explained hereafter.

6. Other circumstances except the wind and the temperature; such as the amount of barometric pressure; the hygrometrical state of moisture or dryness; the actual weather, whether fog, rain, snow, sunshine, &c.; the nature of the sound itself, whether produced by a blow, a gunshot, the voice, a musical instrument; its pitch, quality, intensity; the original direction impressed on the sound, — by turning for instance the muzzle of a gun in one direction or the other; the nature and position of the surface over which the sound is conveyed, whether smooth or rough, horizontal or sloping, moist or dry, &c. do not materially affect the velocity of sound. But many, indeed all, have a very powerful influence on its intensity, or the loudness of the sound, as it reaches the ear from a given distance.

a. Derham found that fogs and falling rain, but especially snow, tend powerfully to obstruct the free propagation of sound, and that the same effect was likewise produced by a coating of fresh fallen snow on the ground, though, when glazed and hardened at the surface by freezing, it had no such influence. Over water, or a surface of ice, sound is propagated with remarkable clearness and strength. Dr. Hutton relates that, on a quiet part of the Thames near Chelsea, he could hear a person read distinctly at 140 feet distance, while on the land the same could only be heard at 76.

Lieutenant Foster, in the third Polar Expedition of Captain Parry, found that he could hold a conversation with a man across the harbour of Port Bowen, a distance of 6696 feet, or about a mile and a quarter. This, however remarkable, falls far short of what is related by Dr. Young, on the authority of Derham, viz., that at Gibraltar, the human voice has been heard ten miles, (perhaps across the Strait.)

Guns fired at Carlscroom were heard across the southern extremity of Sweden as far as Denmark; 80 miles, as Derham states from memory, but according to the map at least 120.

Dr. Hearn, a Swedish physician, relates that he heard guns fired at Stockholm, on the occasion of the death of one of the Royal Family, in 1685, at the distance of 30 Swedish, or 180 British miles.

The cannonade of a sea-fight between the English and Dutch, in 1672, was heard across England as far as Shrewsbury, and even in Wales, a distance of upwards of 200 miles from the scene of action.

b. That sounds of all pitches, and of every quality, travel with equal speed, we have a convincing proof in the performance of a rapid piece of music by a band at a distance. Were there the slightest difference of velocity in the sounds of different notes, they could not reach our ears in the same precise order, and at the exact intervals of time, in which they are played, nor would the component notes of a harmony, in which several sounds of different pitch concur, arrive at once. M. Biot caused several airs to be played on a flute at the end of a pipe 951 metres, or 3120 feet, long, which were distinctly heard by him at the other end, without the slightest derangement in the order or intervals of sequence of the high and low notes.

7. A very material difference is also observed in the intensity with which sounds are propagated, or the distances to which they may be heard with equal distinctness. When they are prevented from spread-

ing and losing themselves in the air, whether by a pipe, by the vicinity of an extensive flat surface, as a wall, or otherwise.

a. This we observe familiarly in speaking pipes conducted from one apartment to another of a building. In the experiments already cited of M. Biot, a person being stationed at one end of the enormous tube above mentioned (which was a combination of cast iron conduit pipes laid down for the supply of Paris with water, forming a continuous canal of equal internal diameter throughout, and having two flexures about the middle of its length), the lowest whisper at one end was distinctly heard at the other, so that, in fact, the only way not to be heard was not to speak at all. Nay, so faithful was the transmission of every agitation of the air, whether sonorous, or otherwise, along the pipe, that a pistol fired at one end actually blew out a candle at the other, and drove out light substances placed there with considerable violence.

At Carisbrook Castle, near Newport, in the Isle of Wight, is a well, 210 feet in depth and 12 in diameter, into which if a pin be dropped, it will be distinctly heard to strike the water. The interior is lined with very smooth masonry.

b. It is evident, without entering into any nice theoretical considerations, that a mechanical impulse of whatever nature impressed on any portion of the air or other medium, whether fluid or solid, and thence communicated to the surrounding parts, if allowed to spread in all directions as from a centre, must reach every more distant point with an energy continually less and less; because the same quantity of motion is communicated in succession to a larger and larger sphere of inert matter; but if only allowed to spread in certain directions, its diminution will be less rapid, in proportion as the quantity of matter successively put in motion increases less rapidly. Hence a sound might be expected to be conveyed with less diminution along a wall than in the open air, the trough or angle between the wall and the ground, in fact, forming two sides of a square pipe, and the divergence of the sound in two directions being thereby in a great measure prevented.

Dr. Hutton relates, that part of the wall of a garden, formerly in the possession of W. Pitt Esq. of Kingston, in Dorsetshire, conveys a whisper in this way nearly 200 feet. It is probably to some such principle that we must refer a fact mentioned by the last named author, which at first sight appears surprising enough. He relates that when a canal of water was laid under the pit floor of the Theatre *Del Argentino*, at Rome, a surprising difference was observed. The voice has since been heard very distinctly when it was before scarcely distinguishable. It is a general remark that sounds are well heard in buildings which stand on arches over water. The cause of this, however, seems to be the echo produced between the water and the arch which unites with, and reinforces, the original sound.

8. When sound in the course of its propagation meets with an obstacle of sufficient extent and regularity, it is reflected, producing the phenomenon we call an echo. A wall, the side of a house, or the surface of a rock, the ceiling, floor, and walls of an apartment, the vaulted roof of a church, all, under proper circumstances, give rise to echos more or less audible. The reflected sound meeting another such obstacle is again reflected, and thus the echo may be repeated many times in succession, becoming, however, fainter at each repetition, until it dies away altogether.

An echo in Woodstock Park (Oxfordshire) repeats 17 syllables by day, and 20 by night. One on the banks of the Lago del Lupo, above the fall of Terni, repeats 15.

In the Abbey Church of St. Alban's is a curious echo. The tick of a watch may be heard from one end of the church to the other. In Gloucester Cathedral, a gallery of an octagonal form conveys a whisper 75 feet across the nave.

An echo on the north side of Shipley Church, in Sussex, repeats 21 syllables.

In the Cathedral of Girgenti, in Sicily, the slightest whisper is borne with perfect distinctness from the great western door to the cornice behind the high altar, a distance of 250 feet. By a most unlucky coincidence, the precise focus of divergence at the former station was chosen for the place of the confessional. — Secrets never intended for the public ear thus became known, to the dismay of the confessors, and the scandal of the people, by the resort of the curious to the opposite point (which seems to have been discovered accidentally), till at length, one listener having had his curiosity somewhat over-gratified by hearing his wife's avowal of her own infidelity, this tell-tale peculiarity became generally known, and the confessional was removed.

In the Whispering Gallery of St. Paul's, London, the faintest sound is faithfully conveyed from one side to the other of the dome, but is not heard at any intermediate point.

In the Manfroni Palace at Venice is a square room about 25 feet high, with a concave roof, in which a person standing in the centre, and stamping gently with his foot on the floor, hears the sound repeated a great many times; but as his position deviates from the centre the reflected sounds grow fainter, and at a short distance wholly cease. The same phenomenon occurs in the large room of the Library of the Museum at Naples.

9. Sound reflected from a plane surface does not spread equally in all directions into the surrounding air; but it has a tendency to confine itself to a direction equally inclined to the reflecting surface with that in which it approached this surface.

Sir John Herschel, assisted by Mr. Babbage, made some experiments with regard to the reflection of sound, in a peculiarly favorable situation beneath the suspension bridge across the Menai strait in Wales, where there is a remarkably fine echo. The sound

of a blow on the middle of one of the piers at *A* (fig. 1.) was returned to the same point from the opposite pier *B*, at a distance of 576 feet. But as the auditor deviated to either side of *A*, the return was proportionably fainter, and was scarcely heard by him when his station was at *C*, a little beyond the extreme edge of the pier; though another person stationed at *C'* (on the same side of the water), at an equal distance from *A*, heard it well.

10. The most favorable position for the production of a distinct echo from plane surfaces is, when the auditor is placed between two such, exactly half way.

In this situation the sounds reverberated from both will reach him at the same instant, and reinforce each other. If nearer to one surface than the other, the one will reach him sooner than the other, and the echo will be double and confused.

11. If the echoing surface be concave towards the auditor, the sounds reflected from its several points will, after reflection, converge towards him, exactly as in the more familiar instance of reflected rays of light; and he will receive a sound more intense than if the surface were plane, and the more so the nearer it approaches to a sphere concentric with himself; the reverse if convex. If the echo of a sound excited at one station be required to be heard most intensely at another, the two stations ought to be *conjugate foci* of the reflecting surface, i. e., such that, if the reflecting surface were polished, rays of light diverging from one would be made after reflection to converge to the other. Hence if a vault be in the form of a hollow ellipsoid of revolution, and a speaker be placed in one focus, his words will be heard by an auditor in the other, as if his ear were close to the other's lips. The

same will hold good if the vault be composed of two segments of paraboloids, having a common axis, and their concavities turned towards each other; only in this case, sounds excited in the focus of one segment will be collected in the focus of the other, after two reflections.

An attention to the doctrine of echos is of some, though we think a rather overrated, importance to the architect in the construction of buildings intended for public speaking or music, especially if they be large. In small buildings the velocity of sound is such, that the dimensions of the building are traversed by the reflected sound in a time too small to admit of the echo being distinguished from the principal sound. In great ones, on the other hand, as in churches, theatres, and concert rooms, the echo is heard after the principal sound has ceased; and if the building be so constructed as to return several echos in very different times, the effect will be unpleasant. It is owing to this that in cathedrals the service is usually read in a sustained uniform tone, rather that of singing than speaking, the voice being thus blended in unison with its echo. A good reader will time his syllables, if possible, so as to make one fall in with the echo of the last, which will thus be merged in the louder sound, and produce less confusion in his delivery. For music, in apartments of moderate size, all objects which can obstruct the free reflection of sound from the walls, floor, and ceiling are injurious. The echo is not sensibly prolonged after the original sound, and therefore only tends to reinforce it, and is of course highly advantageous. In large ones, an echo can only be advantageous in the performance of slow pieces (as church music). The prolongation of a chord, after the harmony is changed, can be productive of nothing but dissonance. When ten notes succeed one another in a second, as is often the case in modern music, the longitudinal echo of a room, 55 feet long, will precisely throw the second reverberation of each note on the principal sound of the following one wherever the auditor be placed; which, in most cases, will produce (in so far as it is heard) only

discord. Much mistake seems to be prevalent on this subject. Thus it is said that the form of an orchestra should be parabolic, &c., and that the rays of sound should be reflected out in parallel lines to the audience. But even if they were so, the reflected sounds could not possibly reach them in the same time with the direct; and in acoustics it is of little moment in what direction sounds reach the ear, which is not, like the eye, capable of appreciating direction with any precision, or collecting the rays or waves of sound to a focus within the ear. It is not possible to place a whole band in the focus of a parabolic or elliptic orchestra, or a whole audience in that of a corresponding opposite segment. We may add, too, that an apartment would be worse lighted, were its internal surface a polished semi-ellipsoid with a candle in the focus, than if it were of the usual shape, and its walls and ceilings a dead white. The object to be aimed at in a concert-room is, not to deafen a favored few, but to fill the whole chamber equally with sound, and yet allow the echo as little power to disturb the principal sound, by a lingering after-twang, as possible. But, whether for music or for oratory, open windows, deep recesses, hangings, or carpeting, and a numerous audience in woollen clothing, are all unfavorable to good hearing. They are to sound, what black spaces in an apartment would be to light; they return back none, or next to none, of what falls on them. Their fault is not so much that they reflect it irregularly, as that they do not reflect it at all.

12. The rolling of thunder and its sudden and capricious bursts and variations of intensity arise partly from the echos among the clouds, and partly from the zigzag course of the lightning.

a. As clouds are collections of particles of water which, however minute, are yet in a liquid state, and therefore each is individually capable of reflecting sound, there is no reason why very loud sounds, like bright lights, should not be reflected confusedly; and that such is the case, has been ascertained by direct observation on the sound of cannon. Messrs. Arago, Matthieu, and Prony,

in their experiments on the velocity of sound, observed, that under a perfectly clear sky, the explosions of their guns were always heard single and sharp; whereas when the sky was overcast, or even when a cloud came in sight over any considerable part of the horizon, they were frequently accompanied with a long continued roll like thunder, and occasionally a double sound would arrive from a single shot.

b. To understand the other source of the irregularities of thunder, we must premise that, *cœteris paribus*, the estimated intensity of a sound is proportional to the quantity of it (if we may so express ourselves) which reaches the ear in a given time. Two blows equally loud, at precisely the same distance from the ear, will sound as one of double the intensity; a hundred, struck in an instant of time, will sound as one blow a hundred times more intense, than if they followed in such slow succession that the ear could appreciate them singly.

Now let us conceive two equal flashes of lightning, each four miles long, both beginning at the same point, but the one running out in a straight line directly away from the auditor; the other describing an arc of a circle having him in its centre. Since the velocity of electricity is incomparably greater than that of sound, the thunder may be regarded as originating at one and the same instant in every point of the course of either flash. But it will reach the ear under very different circumstances in the two cases. In that of the circular flash, the sound from every point will arrive at the same instant, and affect the ear as a single explosion of stunning loudness. In that of the rectilinear flash, on the other hand, the sound from the nearest point will arrive sooner than from those at a greater distance; and those from different points will arrive in succession, occupying altogether a time equal to that required by sound to run over four miles, or about twenty seconds. Thus the same amount of sound is in the latter case distributed uniformly over 20 seconds of time, which in the former arrives at a single burst; of course, it will have the effect of a long roar, diminishing in intensity as it comes from a greater and greater distance. If the flash be inclined in direction, the sound will reach the ear more

compactly (i. e. in shorter time from its commencement), and be propórtionably more intense.

If (as is almost always the case) the flash be zigzag, and composed of broken rectilinear and curvilinear portions, some concave, some convex to the ear; and if, especially, the principal trunk separates into many branches, each breaking its own way through the air, and each becoming a separate source of thunder, all the varieties of that awful sound are easily accounted for.

CHAPTER II.

MATHEMATICAL THEORY OF THE PROPAGATION OF SOUND.

13. A GENERAL notion of the mode in which an impulse communicated to one portion of air, or other elastic fluid, is diffused through the surrounding portions, and successively propagated to portions at a greater and greater distance from the original source of the motion, may be obtained by considering the way in which a tremor runs along a stretched cord, or in which waves, excited in the surface of still water, dilate themselves circularly, and propagate a motion impressed on one point of the surface, in all directions to a distance.

a. In the case of the stretched cord, conceive a blow given to a point in the middle of the cord transversely to its length. The point to which the blow is given will be thrown out of the straight line, and a flexure or angle will be formed in that part. Owing, however, to the inertia of the cord, the displacement of the particles in the first instant will be confined to the immediate neighbourhood

of the point of impulse; so that the cord will not at once asssume the state represented in fig. 2., consisting of two straight portions *AB*, *BC*, forming a very obtuse angle *ABC*; but rather that in fig. 3., in which the greater part on either side *AD*, *EC* retain their original position; and a small part *DBE*, proportioned to the violence and suddenness of the blow, is, as it were, bulged out into an angular form *DBE*. The particle at *B* then is solicited on both sides by the tension of the cord in directions *BD*, *BE*; but these tensions, which in the quiescent state of the string exactly counteracted each other, now only do so in respect of those parts of each which, when resolved, act in directions parallel to *DA*, *EC* respectively. The other resolved portions, perpendicular to them, conspire and urge the point *B* towards its point of departure *b*. As there is no force to counteract this (the impulse being supposed momentary), *B* will obey their solicitation, and approach *b* with an accelerated velocity. But, action and reaction being equal and contrary, the same force by which the molecule *E* drags *B* *down*, will be exerted on *E* to drag it *up*, or out of the line; and it will be the same with the point *D*. There will, then, be an instant when the cord will have assumed the figure *AD′DBEE′C*. But the acquired velocities of the points *B*, *D*, and *E* urge them onward, the first towards the line and the other two from it; and at the next moment the forces are reversed, *B* tending to drag both *D* and *E* down to the line. But the momentum of *B* is expended in the effort, and by the time it has reached its original place in the line, its inertia is destroyed, and it rests there without a tendency to go beyond it on the other side. Meanwhile, however, *D* and *E* have attained their greatest elevation; and thus the protuberance *DBE* is resolved into two *D′DB* and *BEE′* (of less height, however) on either side. In like manner the particles *D* and *E*, in returning to their places, drag up the next adjoining *D′* and *E′*, and then the next, and so on; and thus the summits of the protuberances advance along the line, and correspond in succession to all its points, and the visible effect is an undulation or wave, which runs along the cord with a velocity greater, the greater is the force with which the cord is strained, as it manifestly ought to be, since the

rapidity with which each molecule returns from its displaced situation is greater, as the force urging it is so; and this force is nothing more than the resolved part of the tension.

b. When a wave is excited in water by throwing in a stone, one portion is violently driven down and the surrounding part heaped up above its natural level. The ridge subsides and fills up the vacuity; but as its pressure is the same on both its sides, it also forces up the fluid outside of it, and thus forms an exterior ridge. This new ridge, in subsiding, forms still another ridge; and thus an advancing wave is formed; and the same action, taking place on all sides of the centre, the wave advances equally in every direction.

c. It is by no means intended, in what is here said, to give an accurate account of what passes in either of these cases, (in fact, it is very far from being so, as the reader by a little attention will soon perceive,) but only to give a first conception of the propagation of motion by undulations or waves.

14. A wave is the form, space, or outline, whether linear or superficial, comprehending all the particles of an undulating body which are at once in motion.

In the preceding cases, it must be observed that the wave which advances on the surface of the water — the sinuosity which runs along the stretched cord — are neither of them things, but forms. They are not moving masses advancing in the direction in which they appear to run, but outlines or figures, which at each instant of time include all the particles of the water or the cord which, it is true, are moving, but whose motion is in fact *transverse* to the direction in which the waves advance. But this is by no means an essential condition.

15. The motion of a wave is carefully to be distinguished from that of the particles which it includes, and its velocity is altogether independent of the extent of their excursions from the state of rest.

Waves in a Field of standing Corn.

a. The waves in a field of standing corn, as a gust of wind passes over it, afford a familiar example of the relation between the motion of the wave and that of the particles of the waving body, and of the mutual independence which may subsist between these two motions. The gust in its progress depresses each ear in its own direction, which, so soon as the pressure is removed, not only returns, by its elasticity, to its original upright situation, but by the impetus it has thus acquired, surpasses it, and bends over as much, or nearly as much, on the other side; and so on alternately, oscillating backwards and forwards in equal times, but continually through less and less spaces, till it is reduced to rest by the resistance of the air. Such is the motion of each individual ear; and as the wind passes over all of them in succession, and bends each equally, all their motions are so far similar. But they differ in this, that they commence not at once but successively.

Suppose (to fix our ideas) the wind runs over 100 feet in a second, and that the ears stand one foot asunder, and each makes one complete vibration to and fro in two seconds. Suppose A (fig. 4.) to be the furthest point which the wind at any given instant of time has reached, or the last ear which it has just bent, and let the action of the wind be regarded as lasting only for a single instant. Then will the next preceding ear B have already begun to rise from its bent position, the next C will have risen rather more, and the 50th ear G (since the distance AG is 50 feet, and consequently since $\frac{50}{100} = \frac{1}{4}$ of two seconds have elapsed since the wind was at G) will have gone through one fourth of its complete vibration to and fro, and will have just attained its upright position; so that the ears F, E immediately adjacent towards A will not yet have quite recovered their perpendicularity, but still lean somewhat forwards; while those on the other side H, I will have surpassed the perpendicular, and have begun to sway backwards; consequently at G the stalks will on both sides be convex towards G, and the ears in that place will be further asunder than in their state of rest, and will appear as it were *rarefied* when viewed by a spectator so distant as to take in a great extent at once.

Still further in rear of the wind, as 100 feet, at L, the 100th ear will have swung backwards as far as possible, and will just have its motion destroyed. The preceding stalk, K, will still want something of its extreme backward flexure; the subsequent one, M, will already have risen a little, and therefore the interval of the ears K, N will be just what it was in the state of rest. At L, then, the spectator will see the ears at their natural distances from each other. Again the 150th stalk, Q, in rear of the wind will have had time to rise again erect from its backward inclination, three fourths of two seconds having elapsed since its first bending forward. The 149th, P, will not be quite erected; the 151st will have surpassed the erect state, and have again begun to lean forward. The stalks then on both sides of Q will curve towards Q, and their ears will therefore be closer together than in their natural state, and will appear to be *condensed.*

Finally the 199th, 200th, and 201st ears will be again in the same relative state as the 99th, 100th, and 101st; only leaning forwards instead of backwards, and therefore neither condensed nor rarefied.

The field, then, will present to the spectator a series of alternate condensations and rarefactions of the corn ears, separated by intervals in their natural state of density; and this series will extend so far in rear of the wind, till the resistance of the air and want of perfect elasticity in the stalks shall have reduced them to rest, and these alternations, by the difference of shading they offer, will become apparent to his sight as dark and bright zones.

b. It matters not for our present purpose, that the impulse is, in the case here taken, not propagated mechanically from ear to ear by mutual impulse, but that each moves independently of all the rest. All we want to illustrate is the distinction between the *wave* and the *moving matter*, and the independence of their motions. The waves here run along with the speed of the wind, whatever that may be; for it is always the point 50 feet in rear of the wind that is most rarefied, and that at 150 that is most condensed; and the interval between the 1st and the 200th ear, comprehending ears in every state or phase of their vibrations is what we term a wave. The velocity of the wave is in this case then that of the wind, and

is totally distinct from and independent of that of each or any particular ear. The one is a constant, the other a variable quantity; the one a general resulting phenomenon, the other a particular, individual, mechanical process, going on according to its own laws.

c. Neither is it of the least consequence, whether the excursions of the several stalks from their position of rest be great or little; whether the degree of bending, or force of the wind, be great or small, provided its velocity be constant. In the case of wind, indeed, the force depends on the velocity; but if we conceive the impulse given by a rigid rod, made to sweep across the field, any greater or less degree of flexure might be given, with the same velocity, by a mere change of its level; but the velocity of the wave would still be that of the rod in every case.

16. The breadth of a wave is equal to the space run over by it in a time equal to that in which any molecule of the waving body performs one complete vibration, going and returning, through all the phases of its motion.

a. In the above example, the breadth of the wave was 200 feet, the space run over by the wind and wave in two seconds, the time of one complete vibration of a stalk; and the same result would have been obtained in any other case.

b. In the case here taken, the motion of the individual molecules is not, as in the former instances, transverse to that of the wave, but parallel to it. It is then hardly to be termed a *form* or an *outline.* To such a wave the term *pulse* is often applied. Whatever be the nature of the internal motions, however, the general name wave or undulation will equally apply, and will be used in future indiscriminately for all sorts of propagated impulses. It is not even necessary that the motions of the constituent particles should be rectilinear, or even lie in one plane. We may suppose the impelling cause to be a whirlwind. In this case each ear will have a rotatory or twirling motion, or the stalk a conical one simply, or in addition to its flexure in a vertical plane. But the breadth of the wave is independent of these particularities, and only changes with its velocity and the time of the vibration of the molecules.

17. The problem to investigate the general laws of the propagation of sound is one of the utmost complexity, and has been resolved only under very restricted conditions; enough, however, to verify principal facts, and establish leading points, in the doctrine of Acoustics. We shall be far from attempting to present here any thing approaching to a sketch of the profound geometrical researches which have been bestowed on this department of physics, by Euler, D. Bernouilli, Lagrange, Poisson, &c.; but shall confine ourselves to just so much developement of the mathematical analysis of the subject, as will suffice for the demonstration of the chief theoretical propositions we shall have occasion for in the sequel.

18. It is required to investigate the propagation of sound in a straight canal of equal bore throughout, filled with air or any other elastic fluid of equable density and elasticity, unacted on by gravity, and of which the transverse section is so small, and the sides so perfectly polished, that we may regard the motions of all particles in the same section as exactly similar; so that each section shall merely advance or recede in the pipe, without any lateral change of place of its constituent molecules *inter se.*

a. Let AB (fig. 5.) be such a pipe, and let any section of it, as A, be agitated by an external cause, with any arbitrary motion, i. e. one whose duration and extent, and whose velocity at every instant shall be entirely dependent on the will, or, if we please, the caprice of an external operator sufficiently powerful to command it; and let us inquire how any other section whatever, situated at any assigned distance, x, from A, will move in consequence of this arbitrary motion of A.

b. Let us then conceive, that, in general, the section or stratum of the molecules $a\,a\,b\,b$, whose distance from the initial place A of the section A is represented by x, shall, after the lapse of any time t, have been transported into the situation $\alpha\ \alpha\ \beta\beta$, at a distance $A\,\alpha = y$ from the same fixed point A. Let x' x'', &c. be the distances of the next consecutive sections from the fixed point A, in their state of rest, and y', y'', &c. their distances after the lapse of the same time t. Then will

$$x' - x = d\,x,\ x'' - x' = d\,x',\ x''' - x'' = d\,x'', \text{ \&c.}$$

be the thicknesses (supposed infinitely small) of these strata, or the spaces occupied by them (taking the area of the section for unity) in their quiescent state, and

$$y' - y = d\,y,\ y'' - y' = d\,y',\ y''' - y'' = d\,y'', \text{ \&c.}$$

the same in their state of motion. Now as these strata were in contact at the origin of the motion, and are held together by the pressure of the surrounding fluid, they will remain in contact, and advance and recede along the pipe as one mass, only the space they will occupy at different points of their motion will be variable, according to the degree of condensation or dilatation they may have undergone in virtue of their motion itself. If, for instance, at any moment the hinder of them $d\,y$ be in the act of urging forward the next $d\,y'$, it will be condensed; if retreating, rarefied in comparison with the state of the preceding one $d\,y''$.

c. Now any stratum of molecules $d\,y'$, interjacent between two others $d\,y$ and $d\,y''$, can only undergo a change in its velocity when urged by some force, and the only force which can urge it is the difference of pressures it may experience on its two faces by the difference (if any) of the elasticities of the adjacent strata $d\,y''$ and $d\,y$. If we can estimate this, the laws of Dynamics will enable us to express the consequent change of motion.

To this end, then, let the elasticity of the air in its quiescent state be represented by E, which is a given quantity, and is measured by the weight of a column of mercury sustained by it, or by the length of a homogeneous column of air of the same density, whose weight shall suffice to keep it so compressed, or be

equal to that of the column of mercury in the barometer. Then, since the elasticity of air is inversely as the space it occupies, (*cæteris paribus*),

$$d\,y : d\,x :: \text{elasticity of the air when occupying } d\,x = E$$
$$: \text{its elasticity when occupying } d\,y = E\,\frac{d\,x}{d\,y}.$$

Similarly the elasticities of the air occupying $d\,y'$ and $d\,y''$ are represented by $E\,.\,\frac{d\,x'}{d\,y'}$ and $E\,\frac{d\,x''}{d\,y''}$. Hence the plane separating the strata $d\,y$ and $d\,y'$ will be pressed forward by the elasticity $E\,\frac{d\,x}{d\,y}$, and backward by $E\,.\,\frac{d\,x'}{d\,y'}$. So that it will, on the whole, be urged forward by the force

$$-E\left(\frac{d\,x'}{d\,y'} - \frac{d\,x}{d\,y}\right) = -E\,.\,d\,.\,\frac{d\,x}{d\,y}.$$
$$= +E\,.\,\frac{d\,x\,d^2\,y}{d\,y^2}$$
$$= +E\,.\left(\frac{d\,x}{d\,y}\right)^2\frac{d^2\,y}{d\,x},$$

the differentials being taken on the supposition that $d\,x$ is constant, or that $d\,x$, $d\,x'$ $d\,x''$, &c. are originally equal.

Now if we denote by H the length of a homogeneous column of air necessary to counterbalance the elasticity of the quiescent air, and by D its density, we have

$$HD = \text{its weight} = \text{the elasticity } E,$$
$$D = \frac{E}{H}$$

and

$$d\,x'\,.\,D = d\,x\,.\,D = \text{the weight of the stratum } d\,x',$$
$$= \frac{E}{H}\,.\,d\,x.$$

Thus, then, the moving force $E\left(\frac{d\,x}{d\,y}\right)^2\frac{d^2\,y}{d\,x}$ is exerted in urging forward a weight $= \frac{E}{H}\,.\,d\,x.$

d. Now the distance of the mass thus urged from the fixed point A, at the expiration of the time t, is y'. Hence the velocity of the particle $d\,y'$ or, which comes to the same, of the particle $d\,y$ in contact with it is $\frac{d\,y}{d\,t}$; the increase of velocity during the instant $d\,t$ is $\frac{d^2\,y}{d\,t}$; and we have by Dynamics

$$\frac{E}{H} \cdot d\,x \cdot \frac{d^2\,y}{d\,t^2} = 2\,g\,E\left(\frac{d\,x}{d\,y}\right)^2 \frac{d^2\,y}{d\,x^2}$$

or

$$\left(\frac{d\,y}{d\,x}\right)^2 \frac{d^2\,y}{d\,t^2} = 2\,g\,H\,\frac{d^2\,y}{d\,x^2} \qquad \text{(I.)}$$

where

$$2\,g = 32\,.\,180 \text{ British standard feet,}$$

gravity being the unit of accelerating force, and t being expressed in mean solar seconds; and all linear quantities such as H, x, y being expressed in feet.

e. This is, in fact, an equation of partial differentials, y being at once a function both of x the original distance of the stratum $d\,x$ from the origin of the motion, and of t the time elapsed. In its present form, simple as it appears, it is altogether intractable and incapable of integration. In fact, it embraces a class of dynamical problems of very great complexity; for it is evident that, since no hypothesis has been made in any way limiting the extent of the excursions of the original or subsequent strata from their points of quiescence, this equation must contain the general expression of all possible motions of elastic fluids in narrow pipes, whether great, as when urged by pistons or driven by bellows, or small, as are the tremors which cause sound. In the theory of sound we suppose the agitations of each molecule so minute as not to move it sensibly from its point of rest. Experience confirms this. Sounds transmitted through a smoky or dusty atmosphere cause no visible motion in the smoke or floating dust, unless the source of sound be so near as to produce a *wind*, which, however, is always insensible at

a very moderate distance. If we introduce this condition, the equation (I.) admits of integration; for the whole amount of motion of each molecule being extremely minute, their differences for consecutive molecules, or the amount of the rarefactions and condensations undergone must be much more so. Hence the value of $\frac{d\,y}{d\,x}$ which expresses the ratio of the condensation of the stratum $d\,y$ in motion and in rest, may be regarded as equal to unity, and the equation becomes simply,

$$\frac{d^2\,y}{d\,t^2} = 2\,g\,H\,.\,\frac{d^2\,y}{d\,x^2} \qquad \text{(II.)}$$

which is the equation of sound regarded as propagated in one dimension, that of length, only; or, as prevented from spreading laterally by a pipe.

f. The complete integral of this equation is

$$y = F\,(x + \sqrt{2\,g\,H}\;t) + f\,(x - \sqrt{2\,g\,H}\;t) \qquad \text{(III.)}$$

where F and f denote arbitrary functions of the quantities within the parenthesis, and which must be determined by a consideration of the initial state of the fluid, or by the nature of the motion originally communicated to its molecules.

g. Let us, then, suppose that, at the commencement of the motion, we have impressed on each section of the fluid, along its whole extent, any arbitrary velocities and condensations, by any means whatever, so as to comprehend in our investigation all possible varieties of initial motion, whether expressible by regular analytical functions, or depending on no regular law whatever.

It is manifest that these conditions will be expressed by assuming arbitrary functions of x, such as $\varphi\,(x)$ and $\psi\,(x)$ for the ***initial*** values of the two partial differentials $\frac{dy}{d\,t}$ and $\frac{d\,y}{d\,x}$, whereof the former represents in all cases the velocity (v) of a particle which would be at the distance x from the origin of coördinates in the state of equilibrium, and the latter the linear extent (e) of that particle compared with its original extent, to which its density and elasticity

are reciprocally proportional. Now differentiating (III.) we get for the general values of v and e,

$$v = \frac{dy}{dt} = \sqrt{2gH}\Big(F'(x+\sqrt{2gH}\,t) - f'(x-\sqrt{2gH}\,t)\Big), \quad \text{(IV.)}$$

$$e = \frac{dy}{dx} = F'(x+\sqrt{2gH}\,t) + f'(x-\sqrt{2gH}\,t); \quad \text{(V.)}$$

consequently their initial values, making $t = 0$, will be

$$\varphi(x) = \sqrt{2gH}\,\Big(F'(x) - f'(x)\Big),$$

$$\psi(x) = F'(x) + f'(x);$$

whence we get immediately

$$\left.\begin{aligned} F'(x) &= \frac{1}{2\sqrt{2gH}}\Big(\sqrt{2gH}\,\psi(x) + \varphi(x)\Big), \\ f'(x) &= \frac{1}{2\sqrt{2gH}}\Big(\sqrt{2gH}\,\psi(x) - \varphi(x)\Big); \end{aligned}\right\} \quad \text{(VI.)}$$

and multiplying by dx and integrating

$$F(x) = \frac{1}{2\sqrt{2gH}}\int\Big(\sqrt{2gH}\,\psi(x) + \varphi(x)\Big)\,dx,$$

$$f(x) = \frac{1}{2\sqrt{2gH}}\int\Big(\sqrt{2gH}\,\psi(x) - \varphi(x)\Big)\,dx;$$

and thus the forms of the functions F and f become known when those of φ and ψ are given.

h. The question of the propagation of sound, however, does not require us to concern ourselves with these functions, as a knowledge of the actual velocity and density of any molecule at any instant is sufficient for our purpose. Substituting then in (IV.) and (V.) the values of F' and f' given in (VI.), we get

$$\left.\begin{aligned} v = \frac{\sqrt{2gH}}{2}&\Big(\psi(x+\sqrt{2gH}\,t) - \psi(x-\sqrt{2gH}\,t)\Big) \\ &+ \tfrac{1}{2}\Big(\varphi(x+2\sqrt{2gH}\,t) + \varphi(x-\sqrt{2gH}\,t)\Big), \end{aligned}\right\} \quad \text{(VII.)}$$

$$e = \tfrac{1}{2}\left(\psi\,(x + \sqrt{2gH}\,t) + \psi\,(x - \sqrt{2gH}\,t)\right)$$
$$+ \frac{1}{2\sqrt{2gH}}\left(\varphi\,(x + \sqrt{2gH}\,t) - \varphi\,(x - \sqrt{2gH}\,t)\right); \qquad \text{(VIII.)}$$

or, as it may be written,

$$v = \tfrac{1}{2}\left(\varphi\,(x + \sqrt{2gH}\,t) + \sqrt{2gH}\,\psi\,(x + \sqrt{2gH}\,t)\right)$$
$$+ \tfrac{1}{2}\left(\varphi\,(x - \sqrt{2gH}\,t) - \sqrt{2gH}\,\psi\,(x - \sqrt{2gH}\,t)\right), \qquad \text{(IX.)}$$

$$e = \frac{1}{2\sqrt{2gH}}\left(\varphi\,(x + \sqrt{2gH}\,t) + \sqrt{2gH}\,\psi\,(x + \sqrt{2gH}\,t)\right)$$
$$- \frac{1}{2\sqrt{2gH}}\left(\varphi\,(x - \sqrt{2gH}\,t) - \sqrt{2gH}\,\psi\,(x - \sqrt{2gH}\,t)\right); \qquad \text{(X.)}$$

These are essentially the same expressions with those given by Euler and Poisson, and they comprise the whole theory of the linear propagation of sound.

19. If the preceding formulas are applied to the case of a very small primitive disturbance, they lead to the result that the velocity of sound is uniform, is independent of the nature, extent, and intensity of this disturbance, and is expressed by $\sqrt{2gH} = 916$ feet.

a. If the initial disturbance is small, the initial state of the fluid must consist in a general repose of the whole of an infinitely extended column, except a very small portion which we may suppose agitated by any arbitrary motion. This, in fact, is the simplest case of the production of sound; the primitive disturbance of the air being always confined within extremely small limits compared to the distances to which the sound is propagated.

Let us then conceive the initial disturbance to take place over a minute length $2a$ of the column, whose middle we will suppose to be at A the origin of coördinates. This amounts to supposing that, for every value of x not comprised within the limits

$$x = -a, \quad \text{and} \quad x = +a,$$

we have, for φ and ψ

$$\text{the initial velocity} = \varphi(x) = 0,$$
$$\text{the initial extent of the particle} = \psi(x) = 1;$$

admitting them to have any arbitrary values between these limits.

b. Let us now regard what happens only on the positive side of the origin of x, and consider the motions of the molecule which is at the distance x from this origin, x being greater than a.

Since x is greater than a, $x + \sqrt{2gH} \cdot t$ being greater than x, must also be greater than a, and, therefore, we must have

$$\varphi(x + \sqrt{2gH} \cdot t) = 0$$
$$\psi(x + \sqrt{2gH} \quad t) = 1$$

If, now, the time t is so small that

$$x - \sqrt{2gH} \cdot t > a,$$

or so large that $x - \sqrt{2gH} \cdot t$, becoming negative, we have

$$x - \sqrt{2gH} \cdot t < -a,$$

and, in either case,

$$\varphi(x - \sqrt{2gH} \cdot t) = 0,$$
$$\psi(x - \sqrt{2gH} \cdot t) = 1;$$

and the general values of v and e (VII.) and (VIII.) become

$$v = \frac{\sqrt{2gH}}{2}\left(1 - 1\right) + \tfrac{1}{2}(0 + 0) = 0,$$

$$e = \tfrac{1}{2}(1 + 1) + \frac{1}{2\sqrt{2gH}}(0 - 0) = 1;$$

or the same as in the initial state of the fluid.

The molecule at x remains at rest, then, until a time t, such that

$$x - \sqrt{2gH} \cdot t = a,$$

or

$$t = \frac{x - a}{\sqrt{2\,g\,H}}; \qquad \text{(XI.)}$$

and continues to move for a few instants, until a time t such that

$$x - \sqrt{2\,g\,H} \,.\, t = -\,a,$$

or

$$t = \frac{x + a}{\sqrt{2\,g\,H}}; \qquad \text{(XII.)}$$

when it returns to its quiescent state, where it remains till disturbed by some other cause.

c. The difference between the preceding values of t, (XI.) and (XII.) is the interval, during which the molecule continues to move equal to

$$\frac{x + a}{\sqrt{2\,g\,H}} - \frac{x - a}{\sqrt{2\,g\,H}} = \frac{2\,a}{\sqrt{2\,g\,H}},$$

being the same for each successive molecule since it is independent of x.

d. The time of which another molecule distant x' from A, would commence its motions, is, in the same way,

$$t' = \frac{x' - a}{\sqrt{2\,g\,H}};$$

the difference between which and the time t (XI.) when the molecule at x began to move, is the time which it takes the sound to proceed from one molecule to the other, equal to

$$t' - t = \frac{x' - a}{\sqrt{2\,g\,H}} - \frac{x - a}{\sqrt{2\,g\,H}},$$
$$= \frac{x' - x}{\sqrt{2\,g\,H}}.$$

We have, therefore, for the velocity of the sound,

$$\text{velocity} = \frac{\text{space}}{\text{time}},$$
$$= \frac{x' - x}{t' - t},$$
$$= \sqrt{2\,g\,H}; \qquad \text{(XIII.)}$$

which is independent of a, as well as of the arbitrary functions φ and ψ; so that all sounds travel with the same uniform velocity.

e. The numerical value of $\sqrt{2gH}$ must be obtained in order to compare theory with observation. To this end, if we call $\triangle$ the density of mercury, h the height of the mercury in a barometer exposed to the same pressure as the sounding column, and D the density of the air in it, H being the height of a homogeneous column of such air capable of counterbalancing the elasticity of the sounding fluid, we have

$$D : \triangle :: h : H = h \cdot \frac{\triangle}{D};$$

and, calling V the velocity of sound,

$$V = \sqrt{2gH} = \sqrt{2g\,h \cdot \frac{\triangle}{D}} \,. \qquad \text{(XIV.)}$$

Now, at the freezing temperature, and in a mean state of barometric pressure, we have, according to Biot,

$$2g = 32{\cdot}180 \text{ feet}, \; h = 2.4936 \text{ feet},$$

$$\frac{\triangle}{D} = 10463;$$

so that we obtain

$$V = 916 \text{ feet}.$$

f. The great difference between this result of theory and the velocity of 1090 feet given by observation is too great a discrepancy, to be attributed to any inaccuracy in the determination of the data, which are all of the utmost precision. It is evident, then, that there is something radically insufficient in the theory, as above delivered; and, accordingly, geometers for a long while endeavoured to account for it on various suppositions. The true explanation was reserved for the sagacity of Laplace. But before we state it, it will be necessary to consider what will be the effect of variations of temperature and pressure on the velocity, according to the principles already laid down, and the formula arrived at.

Effect of Temperature.

20. The velocity of sound is not changed by any increase in the density of air caused by pressure.

For at the same time that the density of the air is increased, its elasticity is increased, and precisely in the same ratio; that is, the mass to be moved and the moving force are increased alike, and therefore the accelerating force remains unaltered.

21. The velocity of sound is greater in warm than in cold air, and it increases 0·96 feet * for every additional degree of temperature on Fahrenheit's scale.

a. An increase of temperature tends to dilate the air, and either renders it more elastic in the same space, or more rare with the same elasticity. Hence, on a variation of temperature, the moving force remains unaltered, while the mass to be moved decreases, and therefore an acceleration in all the resulting motions must arise.

b. Let us denote by (D) the density of air in a mean state of barometric pressure h, at the freezing temperature, and by (V) the velocity at this temperature. Since, by the experiments of Gay Lussac, air expands 0.002083 of its volume by every degree of increase of temperature on Fahrenheit's scale, its density under the same pressure at any other temperature τ° above freezing is

$$D = \frac{(D)}{1 + \tau \, . \, 0{\cdot}002083};$$

consequently the expression for the velocity (XIV.) becomes

$$V = \sqrt{2g \, . \, h \, . \, \frac{\Delta}{(D)} \times (1 + \tau \, . \, 0{\cdot}002083)}. \qquad \text{(XV.)}$$

But we have

$$(V) = 916 \text{ feet} = \sqrt{2\,g \, . \, h \, . \, \frac{\Delta}{(D)}};$$

* The value 1·14 feet, which is given in the 5th section, is the same as that used by Herschel; but on examining the calculation, there appears to have been a mistake, which is here corrected, though it is of but little importance.

and therefore

$$\frac{V}{(V)} = \sqrt{1 + \tau \,.\, 0{\cdot}002083},$$

$$= 1 + \tfrac{1}{2}\tau \,.\, 0{\cdot}002083 = 1 + \tau \,.\, 0{\cdot}001042,$$

by neglecting the square, &c. of ·002083 on account of its smallness. Hence

$$\begin{aligned} V &= (V)\,(1 + \tau\, 0{\cdot}001042) \\ &= 916 \text{ feet } (1 + \tau\, 0{\cdot}001042) \qquad \text{(XVI.)} \\ &= 916 \text{ feet} + \tau \,.\, 0{\cdot}96 \text{ feet} \end{aligned}$$

or the velocity increases 0·96 feet for every additional degree of temperature above freezing, and decreases by the same quantity for each degree below freezing.

22. The great difference between the observed velocity of sound and that obtained by theory arises from the developement of heat in the air from the compression which it undergoes.

a. The law of Marriotte, which makes the elastic force of the air proportional to its density, and which has been employed in estimating the elasticity with which each molecule of the aerial column resists condensation, and transmits it to its neighbour, assumes that the temperature of the whole mass of air is alike, and undergoes no change in the act of condensation, and is therefore only true of masses of air which, after compression, are of the same temperature as before. But it is an ascertained fact, that air and all elastic gaseous fluids give out heat in the act of compression, i. e. actually become *hotter*, a part of their latent heat being developed, and acting to raise their temperature. This is rendered evident in the violent and sudden condensation of air by a tight-fitting piston in a cylinder, closed at the end. The cylinder, if of metal, becomes strongly heated; and, if a piece of tinder be enclosed, on withdrawing the piston it is found to have taken fire; thus proving that a heat not merely trifling, but actually that of ignition, has been excited, of at least 1000° of Fahrenheit's scale. Now, when we consider how small

the mass of air in such an experiment is, compared with that of the including vessel, which rapidly carries off the heat generated, it is evident that if air by any cause could be compressed to the same degree without contact of any other body, a very enormous heat would be generated in it. It would, therefore, resist the pressure much more than if cold; and, consequently, would require a much more powerful force to bring it into that state of condensation than, according to Marriotte's law, would be necessary.

Air, then, when suddenly condensed, and out of contact with conducting bodies, resists pressure more (i. e. requires a greater force to condense it equally) than when slowly condensed, and the developed heat carried off by the contact of massive bodies of its original temperature. In other words, it is under such circumstances *more elastic*, and our analytical expression for its elasticity must be modified accordingly. The condensation of the aerial molecules in the production of sound is precisely performed under the circumstances most favorable to give this cause its full influence; the condensation being so momentary that there is no time for any heat to escape by radiation; and the condensed air being in contact with nothing but air, differing infinitesimally from its own temperature; so that conduction is out of the question. Let us see now how this will affect the matter in hand.

b. If we denote by t' the degrees on the scale of Fahrenheit, by which the temperature of the molecules, while in motion and condensed, surpasses that of their quiescent state, the elasticity of the moving molecule is obtained with sufficient accuracy by multiplying $E \cdot \frac{dx}{dy}$ the elasticity before obtained by $1 + t' \cdot 0{\cdot}002083$.

Now whatever be the law according to which the temperature of a mass of air is increased by a sudden diminution of its volume, it is obvious that for very small condensations, such as those considered in the theory of sound, the rise of temperature will be proportional to the increase of density. But the increase of density is $1 - \frac{dy}{dx}$, hence we must have

$$t' = k\left(1 - \frac{dy}{dx}\right)$$

k being a constant coefficient, whose magnitude may become known, either by direct experiment or by the very phenomenon under consideration. The preceding factor becomes, therefore,

$$1 + \tau' \,.\, 0{\cdot}002083 = 1 + k \,.\, 0{\cdot}002083 \left(1 - \frac{dy}{dx}\right),$$

$$= K - k \,.\, 0{\cdot}002083 \frac{dy}{dx},$$

making

$$K = 1 + k \,.\, 0{\cdot}002083;$$

and the elasticity of the condensed molecule is

$$E\frac{dx}{dy}\left(K - k \,.\, 0{\cdot}002083 \frac{dy}{dx}\right) = E\left(K\frac{dx}{dy} - k \,.\, 0{\cdot}002083\right);$$

and the moving force instead of being $-E\,d\,.\,\frac{dx}{dy}$, is

$$-E\,d\,.\left(K\frac{dx}{dy} - k \,.\, 002083\right) = -EK\,d\frac{dx}{dy}.$$

This differs from the expression originally obtained only by the constant factor K. Without, therefore, going again through all the foregoing analysis, we see at once that the general equations of sound will be precisely as before, writing only $K \,.\, H$ for H throughout. Making this change then in the expression for the velocity of sound, it becomes

$$V = \sqrt{2\,g\,HK},$$

$$= \sqrt{2\,g\,H(1 + k \,.\, 0{\cdot}002083)}, \qquad \text{(XVII.)}$$

$$= \sqrt{2\,g\,h\frac{\Delta}{(D)} \,.\, K \,.\, (1 + \tau \,.\, 0{\cdot}002083)}.$$

c. The actual numerical value of the constant coefficient K may be determined, as we have before said, in two ways; either by direct experiment on the increase of temperature developed in a given volume of air by a given condensation, or by a comparison of the formula to which we have arrived with the known velocity of sound. As we have already observed, however, the circumstances under which sound is propagated are far more favorable to the free and full production of the whole effect of the cause in

question than those of any experiments in close vessels. We must not, therefore, be surprised, if the value of K, as derived from such experiments, should differ materially from its value deduced from the velocity of sound; nor *vice versâ*, if the observed velocity of sound should differ materially from that obtained by calculation, from an experimental value of K. It is sufficient, in a philosophic point of view, to have pointed out a really existing cause, a *vera causa*, which *must* act to increase the velocity, and is fully adequate to do so to the extent observed.

d. We have seen that the numerical value of V neglecting K is equal to 916 feet. The observed value on the other hand, is 1090 feet. Hence we have the following equations for determining K and k,

$$\sqrt{2gH} = 916 \text{ feet},$$

$$V = \sqrt{2gH.K} = \sqrt{2gH}\,.\,\sqrt{K},$$

whence we obtain

$$1090 \text{ feet} = 916 \text{ feet} \sqrt{K}$$

$$K = \left(\frac{1090}{916}\right)^2 = 1{\cdot}4 \qquad \text{(XVIII.)}$$

$$k = \frac{K-1}{0{\cdot}002083} = 198.$$

The value of K has also been determined from some ingenious experiments made by Messrs. Clement and Desormes to be

$$K = 1{\cdot}35,$$

whence the velocity at the freezing temperature comes out

$$916 \text{ feet} \sqrt{1{\cdot}35} = 1064 \text{ feet},$$

which falls short of the actually observed velocity only by about 26 feet. M. Poisson has shown that an absorption of heat in the course of the experiments, which might very well happen, would completely reconcile the observed and theoretical velocities.

Laplace, calculating on the experiments of Messrs. Welter and Gay Lussac, has since obtained a still nearer approximation to the theoretical velocity, the difference amounting only to about 10 feet.

In inquiries of such delicacy, and where the effects of minute errors of experiment become so much magnified, it seems hardly candid to desire a more perfect coincidence.

CHAPTER III.

PROPAGATION OF SOUND IN GASES AND VAPORS.

23. THE analysis, by which we have in the foregoing articles determined the laws and velocity of the propagation of sound in air, applies equally, *mutatis mutandis*, to its propagation in all permanently elastic fluids, and in vapors, in so far as their properties are the same as those of gases.

The formula (XVII.) expresses, then, the velocity of sound in all such media, provided for (D) we write instead of the density of atmospheric air that of the gas at the freezing temperature, and under the mean pressure h.

In the case of vapors, we must suppose in calculating the value of (D) that they follow the law of gases in their condensation, and that no portion of them undergoes a change of state to a liquid, by reduction to the standard temperature and pressure.

Suppose the specific gravity of atmospheric air to be denoted by s, and that of any gas or vapor under the same temperature by s'; then if V and V' be the velocitics of sound in air and in the gas or vapor we have

$$V = \sqrt{2\,g\,h\,.\,\frac{\triangle}{s}\,K\,(1 + \tau\,.\,0{\cdot}002083)},$$

$$V' = \sqrt{2\,g\,h\,.\,\frac{\triangle}{s'}\,K'\,(1 + \tau\,.\,0{\cdot}002083)};$$

consequently,

$$\frac{V'}{V} = \sqrt{\frac{s}{s'}\,.\,\frac{K'}{K}}$$

or (XIX.)

$$V' : V :: \sqrt{\frac{K'}{s'}} : \sqrt{\frac{K}{s}};$$

whence the ratios of $s' : s$ and of $K' : K$ being known, the ratio of the velocities is also known.

24. The developement of heat by sudden compression appears to have nearly the same effect upon the velocity of sound in most gases and vapors, which it has in atmospheric air. This effect seems, however, to be very much diminished in the case of hydrogen.

a. If the quantity of heat developed were precisely the same in gases and vapors, which it is in atmospheric air, we should have

$$K' = K,$$

whence, by (XIX.),

$$\frac{V'}{V} = \sqrt{\frac{s}{s'}},$$

or (XX.)

$$V' : V :: \sqrt{s} : \sqrt{s'}.$$

To compare this with experiment directly is impracticable, as no column of any gas, but atmospheric air, can be obtained of sufficient length and purity to determine the velocity of sound in it by direct measure. Indirectly, however, the comparison may be performed by comparing the sounds of one and the same organ-pipe, filled with the gases to be compared, successively, or by other means of a similar kind, of which more hereafter.

Velocity of Sound in various Gases and Vapors.

The following table, which Herschel obtained from a very imperfect and obscure abstract of an inaugural dissertation of M. Van Rees, exhibits the velocities of sound, as deduced from the preceding hypothesis, and compared with experiments instituted by M. Van Rees, in conjunction with Messrs. Frameyer and Moll.

Gas or Vapor.	Velocity of Sound reduced to Freezing.	
	Theory.	Experiment.
	Feet.	Feet.
Oxygen (from Manganese)	1042	1039
Azote, ...	1112	1109
Hydrogen, ...	4047	3000
Carbonic Acid, ...	888	903
Vapor of Water at 154° F.	1347	1213
Vapor of Alcohol at 140° F.	862	949

The differences in the columns probably arise from impurities in the gases, or difficulty in estimating the exact pitch of sounds propagated by them.

b. The preceding determinations are, of course, liable to considerable errors; but the difference between the results of theory and experiment in the case of hydrogen is so great as to warrant a conclusion, otherwise not improbable, that the value of the coefficient K in that gas (at least) is materially different from what it is in others. Experiments are hardly yet sufficiently multiplied to enable us to speak with certainty on this point; but if by any means we are enabled to determine precisely the velocity of sound, in a gas, or indeed in any medium, the ratio of the values of this coefficient in it and in air may be obtained by the following proportion deduced from (XIX.)

$$K : K' :: V^2 . s : V'^2 s'. \qquad \text{(XXI.)}$$

Thus the specific gravity of pure hydrogen being to that of air as 0·0694 : 1, and the velocity of sound in it being to that in air as 3000 to 1090 we have

$$K \text{ in hydrogen} : K \text{ in air} :: (3000)^2 \times 0{\cdot}0694 : (1090)^2 \times 1$$
$$:: \qquad {\cdot}53 \quad : 1$$

so that the value of K in hydrogen would appear to be only half as great as in air, or only about 0·7.

But if K were in reality less than unity, it would follow that heat was absorbed instead of being evolved in the compression of hydrogen, which is an evident absurdity. We cannot even suppose that hydrogen can be compressed without the developement of heat, although the same quantity of heat would raise the temperature of hydrogen only one twelfth part as much as that of air, on account of the difference in their specific capacities for heat. This circumstance would, no doubt, go a great way to diminish the value of K in the case of hydrogen; and we may suppose it to exceed unity by a very small quantity which may be neglected, and we shall have 3470 feet for the velocity of sound in hydrogen.

25. But not only the velocity of sound differs in media of different chemical and mechanical natures. Its intensity, *i. e.* the impression it is capable of producing on our organs of hearing, also varies extremely with a variation in the density of the transmitting medium.

a. This we have already remarked in the case of air, whether rarefied or condensed. Priestly enclosed a piece of clock work, by which a hammer could be made to strike at intervals, in a receiver filled successively with different species of gas. The distances at which sound ceased to be heard were measured. He thus found that in hydrogen the sound was scarcely louder than in a vacuum, (such a one as he could produce). In carbonic acid it was louder than in air, and somewhat louder also in oxygen. Perolle has described some experiments not altogether in agree-

ment with these. The distance at which a given sound ceased to be heard in atmospheric air being 56 feet, he found that in carbonic acid it was 48 only; while in oxygen and nitrous gas the distance was 63, and in hydrogen only 11. Chladni found the sound of hydrogen gas in an organ pipe remarkably feeble and difficult to distinguish, and that of oxygen stronger than that of atmospheric air, but remarked nothing particular in the case of carbonic acid.

b. Leslie relates some very curious experiments, by which it should appear that hydrogen gas is peculiarly indisposed for the conveyance of sound. He rarefied the air of a receiver in which a piece of clock work was enclosed, striking a bell every half minute, 100 times; and then introduced hydrogen gas, when *no augmentation whatever* of the sound took place. Yet more; when the air in the receiver was only half exhausted, and the deficiency filled up with hydrogen gas, not only the sound was not increased, but was actually diminished *so as to become scarcely audible*. If this last fact be correctly stated, (which from the high character of Mr. Leslie, as an experimenter, we must not doubt,) some peculiar modification of the usual process by which sound is propagated must have taken place. It is much to be regretted that the circumstances are not more fully stated; the *pitch* of the bell in air, in the mixed gases, and in hydrogen alone; the dimensions of the receiver; the distances at which the sounds ceased to be heard; and whether the same effect took place when bells of different pitch were struck, and when the bell was muffled, so as to produce no *musical* sound, are all particulars of essential consequence to enable us to form a judgment of what really took place in this interesting experiment.

c. When hydrogen is breathed, which may be done for a short time, but not without inconvenience and even danger, the voice is singularly affected, being rendered extremely feeble, and at the same time raised in pitch.

This is just what ought to arise from the lungs, larynx, and fauces being filled with an exceedingly rare medium; but if, as some experimenters relate, the effect subsists long after the hydrogen

is expired, and the lungs completely cleared of it, this can only be ascribed to some physiological cause depending on its peculiar action on the organs of the voice.

The singular sounds produced by burning this gas in pipes of proper construction have nothing to do with the propagation of sound *in* the gas itself.

CHAPTER IV.

THE PROPAGATION OF SOUND THROUGH LIQUIDS.

26. Liquids are compressible and elastic, and therefore capable of conveying sound.

a. The experiments of Canton, and the more recent ones of Perkins, Oersted, Colladon, and Sturm, have shown that water, alcohol, ether, and, no doubt, all other liquids are compressible and elastic, though requiring a very much greater force to produce a given diminution of bulk than air. Water, according to the experiments of Perkins as computed by Dr. Roget, suffers a condensation of about $\frac{1}{212}$ by a pressure of 100 atmospheres. This result agrees sufficiently well with that of Canton, which gave a condensation of 0·000046 for every atmosphere of pressure, and has since been confirmed by Oersted's researches.

b. Since water, then, and other liquids have the essential property of elastic media, on which the propagation of sound depends, it may be presumed, *à priori*, that sounds are capable of being conveyed by them as well as by the air; and, indeed, better, by reason of their greater density, pursuant to the same law which obtains in gases.

This conclusion is abundantly confirmed by experiment. Hauksbee ascertained that water would transmit a sound excited in air. Anderon describes a number of experiments on the hearing of fishes, from which, indeed, he concludes, that they are altogether devoid of this sense. But a very different conclusion really follows from them. Fishes enclosed in a glass jar appeared (says Anderon) utterly insensible to any sound excited in the air without them, (if unaccompanied with motion) but the slightest tap with the nail on the edge of the jar, although made in such a situation that the motion could not be seen by them, immediately disturbed them. This is easily explicable; and is in fact just what ought to happen. The intensity of sound excited in any medium must evidently be proportioned to the energy of the original impulse, and must therefore be much greater when arising from the direct impact of a solid body on the water, or its containing vessel, than from that of the particles of air in a sonorous wave, whose momentum is necessarily very small. As fishes have no external organs of hearing, sounds must be conveyed to their sensorium by direct propagation through the bones of their heads; and they are probably insensible to, or habitually careless of, those feeble impulses which are communicated from the air.

c. But that the latter impulses do exist, and are audible by our ears, Anderon's paper furnishes proof enough. He made three people, stripped quite naked, dive at once, and remain about two feet below the surface of the water. In this situation he spoke to them as loud as he was able. At their coming up they repeated his words, but said he spoke very low. He caused the same persons to dive about 12 feet below the surface, and discharged a gun over them, which they said they heard, but that the noise was scarce perceivable. He further caused a diver to halloo under water, which he did; and the sound was heard, though faintly. A grenade, exploded about nine feet below the surface, gave a prodigious hollow sound, with a most violent concussion of the earth around. Lastly, he caused a diver to descend with a bell in his hand, whose ringing he (the diver) assured him he could hear distinctly at all depths; adding, also, that he could hear the rush-

ing of water through a flood-gate at 20 feet distance from the place he was in.

The Abbé Nollet, having descended to various depths, from 4 to 24 inches, could hear all sounds made in the air (as a clock striking, a hunter's horn, the human voice, &c.) distinctly, but faint and attenuated.

Franklin, having plunged his head below water, caused a person to strike two stones together beneath the surface; and at more than half a mile distance heard the blows distinctly. These instances are sufficient to show that sound is *audibly* conveyed through water, as well as through air; and, indeed, if properly excited, much better.

27. The velocity of sound in water, as deduced from the experiments most to be relied upon, is about 4708 feet per second at the temperature of 46°·6 Fahrenheit.

A series of experiments on the velocity of sound in sea-water was instituted by M. Beudant, at Marseilles. Two observers, with regulated watches, were stationed in boats at a known distance. Each was accompanied by a diver. A bell was struck at stated intervals at one station; and at the instant of its being heard by the diver at the other he made a signal, and the time was noted by the observer in the boat. Of course, time was lost. The mean result of these observations gives 4921 feet per second for the velocity.

A more careful and no doubt more exact determination was undertaken and executed in 1826, by M. Colladon, in the Lake of Geneva. After trying various means for the production of the sound, as the explosion of gunpowder, blows on anvils, and bells; the latter were preferred, as giving the most instantaneous, and, at the same time, most intense sound, the blow being struck about a yard below the surface by means of a metallic lever. The experiments were all made at night, to avoid the interference of extraneous sounds, and for the better observing of the signals made at

each blow by the flash of gunpowder; the gunpowder was inflamed by the same blow of the hammer by which the bell was struck, so that all loss of time was effectually avoided; and a better spot could not have been chosen, the water being exceedingly deep, without a trace of any current, and of the most transparent purity, and the distances, being from 2 to 9 miles.

By the mean of 44 observations on three different days the velocity was found to be 4708 feet per second. The mean temperature of the water, from trials made at both stations, and half way between, was found to be 46°·6 Fahrenheit.

28. It is required to obtain the analytical expression for the velocity of sound in liquids.

a. To apply the general analysis, by which the velocity of sound in an elastic medium was deduced, to this case, we must express the elasticity in a form somewhat different from that before employed in the case of aerial fluids. For the elasticity of a liquid, or the force which it exerts in opposition to compression, is not proportionate to its density, but to the increase of its density above the natural state; or rather, to the diminution of its bulk by compression below the natural state.

Let us, then, put e for the *compressibility* of a liquid, or, the diminution of bulk it will sustain by an additional pressure of a single atmosphere; h for the standard height of mercury in the barometer; $\triangle$ for the density of mercury at the freezing temperature; and use, fig. 5, the notation of article 18. Then we have, by the above law of elasticity,

the diminution of bulk by the pressure of one atmosphere $= e$

: the compressing force of an atmosphere $= h \,.\, \triangle$

: : the diminution of bulk of the molecule when at $d\,y = 1 - \dfrac{d\,y}{d\,x}$

: the elasticity at $d\,y = \dfrac{h \,.\, \triangle}{e}\left(1 - \dfrac{dy}{dx}\right)$.

Similarly the elasticity at $d\,y' = \dfrac{h \,.\, \triangle}{e}\left(1 - \dfrac{d\,y'}{d\,x'}\right)$.

So that the plane separating $d\,y$ and $d\,y'$ is pressed forward by the difference of these elasticities or by the force

$$\frac{h \,.\, \triangle}{e}\left(\frac{d\,y'}{d\,x'} - \frac{d\,y}{d\,x}\right) = \frac{h\,.\,\triangle}{e}\, d\,.\,\frac{d\,y}{d\,x},$$
$$= \frac{h\,.\,\triangle}{e}\,\frac{d^2\,y}{d\,x}.$$

But, D being the density of the liquid, we have

the weight of the stratum $d\,x' = D\,.\,d\,x$,

which is the weight to be moved by the preceding force; and, consequently, the equation of motion is

$$\frac{d^2\,y}{d\,t^2} = 2\,g\,.\,\frac{h}{e}\,.\,\frac{\triangle}{D}\,.\,\frac{d^2\,y}{d\,x^2};$$

b. As the preceding equation differs from (II.) only in the coefficient of the second member, we can, by simply changing H into $\frac{h}{e}\,.\,\frac{\triangle}{D}$, deduce the velocity of sound in liquids from its velocity in air; and this change, being made in (XVII.) gives us

$$V = \sqrt{2\,g\,.\,\frac{h}{e}\,.\,\frac{\triangle}{D}\,.\,K.} \qquad \text{(XXII.)}$$

29. When the analysis is applied to the case of water, it leads to the result that the velocity of sound in water is not sensibly affected by the developement of heat. And it is most probable that the same thing is true in the case of other liquids.

a. To compare the results of theory with observation in the case of water, we will take the data afforded by the experiments of Messrs. Colladon and Sturm on the water of the Lake of Geneva, in which their observations on the velocity of sound were made. The foreign contents of this water, as appears by the analysis of M. Tingry, amount only to $\frac{1}{6000}$ of its weight, and it may therefore be regarded as pure water, (though, of course, saturated with air.) The compressibility, as obtained from them, is

$$e = 0{\cdot}000049589.$$

Developement of Heat in Water.

The specific gravity of the water of the lake, at the temperature of the experiment, was found to be exactly that of distilled water at its maximum density, the trifling expansion due to the excess of temperature being exactly counterbalanced by the superior density due to the saline contents, so that

$$D = 1.$$

We have, moreover,

$$2\,g = 32{\cdot}180 \text{ feet}, \quad h = 2{\cdot}4936 \text{ feet},$$
$$\triangle = 13{\cdot}568\,;$$

which, substituted in (XXII.), give

$$V = 4685 \text{ feet} \times \sqrt{K}\,;$$

and, if no heat is developed,

$$V = 4685 \text{ feet},$$

which differs from the observed velocity of 4708 feet by only 23 feet, a space run over by the aqueous pulse in one 200th of a second. We may, therefore, conclude that the heat, developed by compression in water, and the consequent increased resistance to sudden condensation, is insensible.

b. From all direct experiments hitherto made, it appears that in water, and all other liquids, the quantity of heat developed by compression is altogether insensible, or at least very minute. So that, most probably, in other liquids, as well as in water, the velocity of sound may be correctly computed when its compressibility is known, without the necessity of having any regard to the developement of heat.

30. It is probable that the waves of sound, like those of light, in passing from a denser to a rarer medium, undergo, at a certain acuteness of incidence, a total reflection.

In the experiments above given of M. Colladon, he observed that although the sound of a blow was well heard directly above

the bell, yet the intensity of the sound so propagated into the air, diminished with great rapidity as the observer removed from its immediate neighbourhood, and at two or three hundred yards it could no longer be heard at all. This fact renders it probable that the sound is reflected from the surface of the water at a certain acuteness of incidence, and ceases to penetrate into the air; so that the sound heard beyond that limit is merely that which diverges *in the air*, from the point immediately above the bell.

M. Colladon was, therefore, obliged to resort to the following ingenious expedient, to render the sounds, excited at a great distance, audible to an observer out of water; and it is only in this situation that any observations on the velocity of sound, worthy of confidence, can be made. He plunged vertically into the water a thin tin cylinder, about three yards long, and eight inches in diameter, closed at the lower end, and open to the air above; thus forming an artificial surface on which the sonorous waves, impinging perpendicularly, might enter the air, and be thence propagated freely as from a new origin; just as we may look into water at any obliquity by using a hollow tube with a glass plate at the end perpendicular to the axis. This contrivance succeeded completely, and he was enabled by its aid to hear the strokes of a bell under water, at a distance of 9 miles, across the whole breadth of the Lake of Geneva, from Rolle to Thonon.

31. The nature of sounds is often very much changed by travelling through a great extent of water, their duration being shortened, and their musical character diminished.

In the same experiments, M. Colladon was led to remark some very curious particulars respecting the nature, intensity, and duration of sounds propagated by water. He observed, first, that the sound of a bell struck under water, when heard at a distance, has no resemblance to its sound in air. Instead of a continued tone, a short sharp sound is heard like two knife-blades struck together. The effect produced by hearing such a short dry sound, at a dis-

tance of many miles from its origin, he compares to that of seeing, for the first time, very distant objects sharply defined in a telescope. When tried at different distances, it preserved this character, varying only in intensity, so as to render it impossible to distinguish whether the sound heard arose from a violent blow at a great distance, or a gentle one near at hand. It was only when within about a furlong, that the musical tone of the bell was distinguishable after the blow. In air the contrary takes place, as every one knows; the shock of the first impulse of the hammer being heard only in the immediate neighbourhood of the bell, while the continued sound is the only one that affects the hearing at a distance. The reason of this curious difference will be apparent when we come to speak of musical sounds.

32. Sounds do not spread round interposed obstacles in water, with the same facility as in air, but are almost wholly cut off by them. Thus the phenomena of sound in water approximate in this respect to the rectilinear propagation of light, and may lead us to presume, that in a medium incomparably more elastic than water, the *shadow* would be still more perfect and more sharply defined. A material support is thus afforded to the undulatory doctrine of light, against one of its earliest and strongest objections — the existence of shadows.

Sounds in air spread round obstacles with great facility, so that by a hearer situated behind a projecting wall, or the corner of a building, sounds excited beyond it are heard with little diminution of intensity. But in water this is far from being the case. When M. Colladon's tin cylinder, or hearing-pipe, already mentioned, was plunged into the water, at a place screened from rectilinear communication with the bell, by a projecting wall running out from the shore, whose top rose above the water, he assures us, that a very remarkable diminution of intensity in the sound was per-

ceived, when compared with that heard at a point very near the former, but within reach of direct communication with the bell; or, so to speak, out of the acoustic shadow of the wall.

CHAPTER V.

PROPAGATION OF SOUND IN SOLIDS AND IN MIXED MEDIA.

33. Solids, if elastic, are equally adapted for the conveyance of sound with fluids, and even better.

By elasticity in a solid is not meant a power of undergoing *great* extensions and compressions, after the manner of air, or Indian rubber, and returning readily to its former dimensions; but rather what is commonly called hardness, in contradistinction to toughness, a violent resistance to the displacement of its molecules *inter se* in all directions. Thus the hardest solids are, generally speaking, the most elastic, as glass, steel, and the hard brittle alloys of copper and tin, of which mirrors are made; and in proportion as they are so, they are adapted to the free propagation of sound through their substance.

34. An important condition in the constitution of solids is homogeneity of substance; and in a substance perfectly homogeneous, we may add, too, uniformity of structure.

a. The effect of want of homogeneity in a medium, on its power of propagating sound, is precisely analogous to that of the same cause in obstructing the free passage of light, and (as the undulatory doctrine of light teaches) for the very same reason. The

sonorous pulses, in their passage through it, are at every instant changing their medium. Now, at every change of medium, two things happen; first, a portion of the wave is reflected, and the intensity of the transmitted part is thereby diminished; secondly, the direction of propagation of the transmitted part is changed, and the sonorous rays, like those of light, are turned aside from their direct course. Thus the general wave is broken up into a multitude of non-coincident waves, emanating from different origins, and crossing and interfering with each other in all directions. Now, whenever this takes place, a mutual destruction of the waves, to a greater or less extent, arises, and the sound is stifled and obstructed.

b. Further yet; as the parts of a non-homogeneous medium differ in elasticity, the velocities with which they are traversed by the sonorous pulses also differ; and thus, among the waves which do ultimately arrive at the same destination in the same direction, some will arrive sooner, some later. These, by the law of interference, tend mutually to destroy or neutralize each other.

c. But of all causes, which obstruct the propagation of sound, one of the most effective is a want of perfect adhesion at the junctures of the parts of which such a medium consists. The effect of this may be conceived, by regarding the superficial strata of molecules of each medium when in contact, as forming together a thin film of less elasticity than either; at which, therefore, a proportionally greater reflection of the wave will take place than if the cohesion were perfect, just as light is much more obstructed by a tissue of cracks pervading a piece of glass, than it would be by any inequality in the composition of the glass itself.

d. A pleasing example of the stifling and obstruction of the pulses propagated through a medium, from the effect of its non-homogeneity, may be seen by filling a tall glass, as a Champagne glass, half full of that sparkling liquid. As long as its effervescence lasts, and the wine is full of air-bubbles, the glass cannot be made to *ring* by a stroke on its edge, but gives a dead, puffy, disagreeable sound. As the effervescence subsides, the tone becomes clearer,

and when the liquid is perfectly tranquil, the glass rings as usual; but on reëxciting the bubbles by agitation, the musical tone again disappears.

To understand the reason of this, we must consider what passes in the communication of vibrations through the liquid, from one side of the glass to the other. The glass and contained liquid, to give a musical tone, must vibrate regularly in unison as a system; and it is clear, that if any considerable part of a system be unsusceptible of regular vibration, the whole must be so. This neat experiment seems to have been originally made by Chladni.

35. To the greater want of homogeneity in the air by day than by night may, in part, be attributed the greater audibility of sounds at the latter time, and, in part, to the stillness of the night.

a. The preceding experiment has been employed by Humboldt to illustrate this familiar phenomenon. He attributes it to the uniformity of temperature in the atmosphere by night, when upward currents of air, heated by their contact with the earth, under the influence of the sun's rays, are no longer continually mixing the upper with the lower strata, and disturbing the equilibrium of temperature. It is obvious that sound, as well as light, must be obstructed, stifled, and dissipated from its one original direction, by the mixture of air of different temperatures (and consequently elasticities) and thus the same cause which produces that extreme transparency of air at night, which astronomers only fully appreciate, renders it also more permeable to sound.

b. There is no doubt, however, that the universal and dead silence generally prevalent at night renders our auditory nerves sensible to impressions, which would otherwise escape them. The analogy between sound and light is perfect in this as in so many other respects. In the general light of day the stars disappear. In the continual hum of noises which is always going on by day, and which reach us from all quarters, and never leave the ear time

to attain complete tranquillity, those feeble sounds which catch our attention at night make no impression. The ear, like the eye, requires long and perfect repose to attain its utmost sensibility.

36. The singular effect of hydrogen gas when mixed with air, already described, art. 25, in unfitting it for the free propagation of sound, may possibly arise from the want of homogeneity in the mixture of gases of different elasticities ; and perhaps the effects of aqueous vapor may be attributed to the same cause.

a. Chemists maintain that, when gases are mixed, the molecules of each form separate and independent systems, being mutually inelastic, and each sustaining a part of the pressure proportional to its own density. They admit, however, that the molecules of one gas (A) act as obstacles to obstruct the free motion of those of another (H) ; and on this principle they explain the *slow* mixture of two gases in separate vessels communicating by a narrow aperture. Granting these postulates, let us conceive a pulse excited in a mixture of equal volumes of two gases. If the velocity of sound in both be alike, the pulse will run on in each, although independently, yet with the same speed, and, at any instant or at any point of the medium, the contiguous molecules of both gases will be moving in the same direction and with the same velocity. They will, therefore, offer no mechanical obstruction to each other's motion, and sound will be freely propagated.

But if they differ in their specific elasticity, the case will be altered. Each being non-elastic to the other, two distinct pulses will be propagated, and will run on with different velocities; the molecules of either gas, at different points beginning and ceasing to be agitated with the pulsation at different instants. Thus an internal motion, a change of relative position among the molecules of the gas (H) and those of the gas (A) will take place, the one set being obliged to force themselves a passage between the other ; in which, of course, a portion of their motion will be diverted in all sorts of lateral directions, and will be mutually destroyed. It

is evident that the greater the difference of specific elasticities, the greater will be the effect of this cause. In hydrogen the velocity of the pulse is nearly three times its velocity in atmospheric air; and of course, it may be expected in this case to act with great efficacy. In azote and oxygen, on the contrary, the velocities are so nearly alike, that very little obstruction can arise from its influence; so that, in so far as the phenomena of sound are concerned, atmospheric air may be looked upon as a homogeneous medium.

b. If saturated with aqueous vapor at high temperatures, however, it is possible that the effect may become sensible, and, perhaps, to this cause may be attributed a phenomenon, mentioned by more than one experimenter on this branch of physics, of the occasional duplication of the sound of a gunshot heard from a great distance, a part of the sound being transmitted quicker than the rest by aqueous vapor, or even by water in the liquid state, suspended in the air. If this be the case, sounds might be expected to be heard double in thick fogs, or in a snow-storm. But the remarkable obstruction to sound caused by fog, and especially by snow, would probably prevent any sound from being heard far enough to permit the interval of the two pulses to be distinguishable.

37. The well known effect of carpeting, or woollen cloth of any kind, in deadening the sound of music in an apartment, arises from the intermixture of air and solid fibres in the carpets through which the sound has to pass, deadening the echo between the ceiling and floor by which the original sound is swelled.

38. A phenomenon noticed by every traveller who visits the Solfaterra near Naples, but whose true nature has been much misconceived, is easily explicable on the principles of art. 34.

The Solfaterra is an amphitheatre, or extinct crater, surrounded by hills of lava, in a rapid state of decomposition by the action of

acid vapors issuing from one principal and many subordinate vents and cracks. The whole soil of the level at its bottom consists of this decomposed lava, whose disintegration, however, is not so complete as to reduce it to powder; but leaves it in coherent white masses of a very loose friable structure. At a particular spot, a large stone violently thrown against the soil, is observed to produce a peculiar hollow sound, as if some great vault were below. Accordingly it is usually cited as a proof of the existence of some vast cavity below, communicating with the ancient vent of the volcano, and perhaps with subterraneous fires; while others ascribe it to a reverberation from the surrounding hills, with which it is nearly concentric; and others to a variety of causes more or less fanciful.

It seems most probable, however, that the hollow reverberation is nothing more than an assemblage of partial echos, arising from the reflection of successive portions of the original impulse in its progress through the soil at the innumerable half-coherent surfaces composing it. Were the whole soil a mass of sand, these reflections would be so strong and frequent as to destroy the whole impulse in too short an interval to allow of a distinguishable after-sound. It is a case analogous to that of a strong light thrown into a milky medium, or smoky atmosphere; the whole medium appears to shine with a nebulous undefined light. This is to the eye, what such a hollow sound is to the ear.

39. The general principle, on which the conveyance of sound through solids depends, is precisely the same as in fluids; and the same formula may be used to express its velocity, when the specific elasticity is known. There are, however, two very important particulars in which they differ.

40. First, the molecules of fluids are capable of displacement *inter se*. Those of solids, on the other hand, are subjected to the condition of never changing their order of arrangement.

Hence arise a multitude of modifying causes, which must necessarily affect the propagation of sonorous pulses through solids, which have no place in fluids, and modes of vibration become possible in the former, which it is difficult to conceive in the latter, whose parts have no lateral adhesion.

Thus we may conceive pulses propagated in solids, like those of a cord vibrating transversely, in which the motion of each molecule is transverse, or oblique, to the direction in which the general pulse is advancing.

41. Secondly, each molecule of a fluid is similarly related to those around it in all directions; in solids, each molecule has distinct sides, and different relations to space, and to the surrounding particles.

a. The cohesion of the molecules of crystallized bodies is different on their different sides, as their greater facility of cleavage in some directions than in others indisputably proves. They must, in consequence, have unequal elasticities in different directions; and thus the velocity of the pulse propagated through a crystallized solid will depend on its direction with respect to the axes of crystallization.

Interruptions of crystalline structure must also produce an effect on the conveyance of sound analogous to that of the mixture of extraneous matter in a medium.

b. Among uncrystallized solids, too, there are many, such as wood, whalebone, &c. which have a fibrous structure, in virtue of which, it is evident, they are very differently adapted to convey an impulse longitudinally and transversely.

The conducting power of wood along the grain is, certainly, very surprising. A simple experiment will show it. Let any one apply his ear close to one end of the longest stick of sound timber, and let an assistant at the other end scratch with the point of a pin, or

tap so lightly with its head as to be inaudible to himself. Every scratch or tap will be distinctly, nay loudly, heard at the other end, as if close to the head.

42. Almost all solids which are tolerably compact conduct sound well, and transmit it with great rapidity. Thus the velocity of sound in cast iron at the temperature of 51°.8 Fahrenheit, is 11090 feet per second; and it is about as much in most of the other metals, and in different kinds of wood.

a. Chladni relates an experiment made by Messrs. Herhold and Rafn, in Denmark, where a metallic wire 600 feet long was stretched horizontally. At one end a plate of sonorous metal was suspended, and slightly struck; an auditor placed at the other, and holding the wire in his teeth, heard at every blow two distinct sounds; the first transmitted almost instantaneously by the metal, the other arriving later through the air.

Messrs. Hassenfratz and Gay Lussac made a similar experiment in the quarries at Paris; a blow of a hammer against the rock produced two sounds, which separated in their progress; that propagated through the stone arriving almost instantly, while the sound conveyed by the air lagged behind. The same thing has been observed in the blasting of rocks in the deep mines of Cornwall.

These experiments were, however, made at intervals too short to give any numerical estimate of the velocity of transmission of sound in the iron or stone.

b. The only direct experiments we have on this subject are those of M. Biot, who, assisted by Messrs. Bouvard, Malus, and Martin, ascertained the interval required for the sound of a blow on the cast iron conduit pipe already spoken of (art. 7.) to traverse measured lengths of it. The pipe consisted of joints of cast iron, each 8.25 feet long, and connected by flanches with collars of lead covered with tarred cloth interposed, and strongly screwed home; each collar measured 0.47 feet. A blow being struck at one end

and heard at the other, the interval between the arrival of the sound through the air and through the iron was noted. The length being known, the time required for the transmission of the aerial sound became known with great precision, and thence the time of transmission through the iron became known also. The mean result of their observations was 11090 feet per second for the velocity of sound in cast iron at the temperature of the experiment, or 51°.8 Fahrenheit. This is about 10½ times its velocity in air.

c. Chladni assigns 11802 feet for the velocity of sound in brass. Laplace, calculating on an experiment of Borda, on the compressibility of brass, makes it 11682 feet.

According to Chladni, the following are the velocities of sound in different solids, that in air being taken for unity: tin = 7½, silver = 9, copper = 12, iron = 17, glass = 17, baked clay = 10 to 12, wood of various species = 11 to 17. The error in the case of iron throws a doubt on all the rest; unless, perhaps, steel be meant.

43. The force of a pull, push, or blow must be transmitted, by an iron bar or chain, with the same velocity as sound. For every 11090 feet of distance, it will, then, reach its point of action one second after the moment of its emanation from the first mover.

In all moderate distances, then, the interval is utterly insensible. But were the sun and the earth connected by an iron bar, no less than 1074 days, or nearly three years, must elapse before a force applied at the sun could reach the earth. The force actually exerted by their mutual gravity may be proved to require no appreciable time for its transmission. How wonderful is this connection!

CHAPTER VI.

THE DIVERGENCE AND DECAY OF SOUND.

44. HITHERTO we have taken no account of the lateral divergence of sound, which we have supposed confined by a pipe; but it is evident that condensation taking place in any section of such a channel will urge the contained air laterally against the side of the pipe, as well as forward along its axis; and, consequently, if the pipe were cut off at any point, the sound would diverge from that point into the surrounding air. Accordingly, when any one speaks through a long straight tube the voice is heard laterally, as if proceeding from the mouth of the speaker at the orifice.

45. Sounds excited in, or impulses communicated to, any portion of the air or other elastic medium, spread, more or less perfectly, in all directions, in space.

a. We say more or less perfectly; for though there are sounds, as the blow of a hammer, the explosion of gunpowder, &c. which spread equally in all directions, yet there are others which are far from being in that predicament.

For instance, a common tuning-fork (a piece of steel in the shape represented in fig. 6.) being struck sharply, when held by the handle (*A*) against a substance, is set in vibration, the two branches of the fork alternately approaching to and receding from each other. Each of them, consequently, sets the air in vibration, and a musical tone is produced. But this sound is very unequally

audible in different directions. If the axis of the fork, or the line to which it is symmetrical, be held upright about a foot from the ear, and if it be turned round this axis while vibrating, at every quarter revolution the sound will become so faint as scarcely to be heard, while in the intermediate arcs of rotation it is heard clear and strong. The audible situations lie in lines perpendicular and parallel to the flat faces of the fork, the inaudible at 45° inclined to them. This elegant experiment, due originally to Dr. Young, has recently been called into notice by Weber.

b. The non-uniformity of the divergent pulses, which constitute certain sounds, is easily demonstrated by considering what happens when a small disc is moved to and fro in a line perpendicular to its surface. The aerial molecules in front of the disc are necessarily in an opposite state of motion from those similarly situated behind it. Hence, if we conceive a wave propagated spherically all around it, the vertices of the two hemispheres in front and behind are in opposite motions with respect to the centre.

But with regard to that wave of the sphere where the vibrating plate prolonged cuts it, there is evidently no reason why *its* molecules should approach to or recede from the centre, or, rather, there is as much reason for one as for the other. They will, therefore, either remain at rest, or move tangentially; so that the motion of the whole sounding surface, or wave, will, in this case, be rather as in fig. 7 than in fig. 8; and a corresponding difference, both in the intensity and character of the sound heard in different directions, may be fairly expected.

46. The mathematical theory of such pulses as these is of the utmost complication and difficulty, depending on the integration of partial differential equations with four independent variables, *viz.*, the time and the three coördinates of the moving molecules. It is therefore of much too high a nature to have any place in an essay like the present. We shall merely content ourselves with stating the following, as general results in which mathematicians are agreed.

1st. The velocity of propagation of a sonorous pulse is the same, whether we regard it as propagated in one, two, or three dimensions, i. e. in a pipe, a lamina, or a mass of air.

2d. Sounds propagated in a free mass of air diminish in intensity, as they recede further from the sonorous centre, and their energy is in the inverse duplicate ratio of this distance, *cæteris paribus;* or, more generally, they are proportional to the *vis viva* of the impinging molecules.

a. We shall not attempt a proof of these propositions in the general cases, but content ourselves with illustrating them in one particular but important case, *viz.* when the initial impulse is confined to a very small space, and consists in any small radiant motion of all the particles of a spherical surface in all directions equally from the centre.

b. Since the initial wave is spherical, and similar in all its parts, it will evidently retain this property as it dilates by the progress of the impulse. If, then, it be conceived to be divided into its infinitesimal elements by a system of pyramidally disposed plane surfaces, having the common vertex in the centre of the sphere, each of these elements will form the base of one of the pyramids, and its molecules will advance and recede along the axis, as the pulse traverses them, without any change of their relative positions, *inter se;* so that the whole wave may be regarded as broken up into partial waves, each advancing as if confined within a pyramidal pipe, independently of all the rest.

c. Now in any one of these imaginary pipes the pulse will be propagated from layer to layer of the included particles, with the same velocity as if the pipe were cylindrical. For the divergence of the sides of the pipe can only cause a lateral extension, and thence a diminished thickness of the stratum, and will, therefore, alter the *velocity* of each molecule and the extent and law of mo-

PART II.

MUSICAL SOUNDS.

CHAPTER I.

THE NATURE AND PRODUCTION OF MUSICAL SOUNDS.

47. EVERY impulse mechanically communicated to the air, or other sonorous medium, is propagated onward by its elasticity, as a wave or pulse ; but, in order that it shall affect the ear, as an audible sound, a certain force and suddenness is necessary.

The slow waving of the hand through the air is noiseless, but the sudden displacement and collapse of a portion of that medium by the lash of a whip produces the effect of an explosion. It is evident that the impression conveyed to the ear will depend entirely on the nature and law of the original impulse, which being completely arbitrary, both in duration, violence, and character, will account for all the variety we observe in the continuance, loudness, and quality of sounds. The auditory nerves, by a delicacy of mechanism, of which we can form no conception, appear capable of analyzing every pulsation of the air, and appreciating immediately the law of motion of the particles in contact with the ear. Hence all the qualities we distinguish in sounds — grave or acute, smooth, harsh, mellow, and all the nameless and fleeting peculiarities, which constitute the differences between the tones of different musical instruments — bells, flutes, cords, &c., and between the voices of different individuals or different animals.

Noise, as distinguished from Musical Sound.

48. Every irregular impulse communicated to the air produces what we call a *noise*, in contradistinction to a musical sound.

If the impulse be short and single, we hear a crack, bounce, or explosion ; yet it is worthy of remark, as a proof of the extreme sensibility of the ear, that the most short and sudden noise has its peculiar character. The crack of a whip, the blow of a hammer on a stone, and the report of a pistol, are perfectly distinguishable from each other.

If the impulse be of sensible duration and very irregular, we hear a crash ; if long and interrupted, a rattle or a rumble, according as its parts are less or more continuous, and so for other varieties of noise.

49. The ear, like the eye, retains for a moment of time, after the impulse on it has ceased, a perception of excitement. In consequence, if a sudden and short impulse be repeated beyond a certain degree of quickness, the ear loses the intervals of silence and the sound appears continuous. The frequency of repetition necessary for the production of a continued sound from single impulses is, probably, not less than sixteen times in a second, though the limit would appear to differ in different ears.

50. If a succession of impulses occur at exactly equal intervals of time, and if all the impulses be exactly similar in duration, intensity, and law, the sound produced is perfectly uniform and sustained, and has that peculiar and pleasing character to which we apply the term musical.

In musical sounds there are three principal points of distinction, the pitch, the intensity, and the quality.

Of these, the intensity depends on the violence of the impulses, the quality on their greater or less abruptness, or, generally, on the law which regulates the excursions of the molecules of air originally set in motion.

The pitch is determined solely by the frequency of repetition of the impulse, so that all sounds, whatever be their loudness or quality, in which the elementary impulses occur with the same frequency, are at once pronounced by the ear to have the *same pitch*, or to be *in unison*. It is the pitch only of musical sounds whose theory is susceptible of exact reasoning, and on this the whole doctrine of harmonics is founded. Of their qualities and the molecular agitations, on which they depend, we know too little to subject them to any distinct theoretical discussion.

51. The means by which a series of equidistant impulses, or, to speak more generally, by which an initial impulse of a periodical nature, (i. e. capable of being represented by a periodical function,) can be produced mechanically, are extremely various.

a. Thus, if a toothed wheel be turned round with uniform velocity, and a steel spring be made to bear against its circumference with a constant pressure, each tooth, as it passes, will receive an equal blow from the spring, and the number of such blows per second will be known, if the velocity of rotation and number of teeth in the wheel be known.

b. The late Professor Robison devised an instrument, in which a current of air passing through a pipe was alternately intercepted and permitted to pass by the opening and shutting of a valve or stopcock. When this was performed with sufficient frequency,

(which could only be done, we presume, by giving a rapid rotatory motion to the stop cock by wheelwork,) a musical tone was produced, whose pitch became more acute as the alternations became more frequent.

This is precisely the principle of the sirene of the Baron Cagniard de la Tour. In this elegant instrument the wind of a bellows is emitted through a small aperture, before which revolves a circular disc, pierced with a certain number of holes, arranged in a circle concentric with the axis of rotation, exactly equidistant from each other, and of the same size, &c. The orifice, through which the air passes, is so situated, that each of these holes, during the rotation of the disc, shall pass over it and let through the air; but the disc is made to revolve so near the orifice, that in the intervals between the holes it shall act as a cover and intercept the air. If the holes be pierced obliquely, the action of the current of air alone will set the disc in motion; if perpendicular to the surface, the disc must be moved by wheelwork, by means of which its velocity of rotation is easily regulated, and the number of impulses may be exactly counted. The sound produced is clear and sweet, like the human voice.

If, instead of a single aperture for transmitting the air, there be several, so disposed in a circle of equal dimension with that in which the holes of the disc are situated, that each shall be opposite one corresponding hole when at rest, these will all form sounds of one *pitch*, and being heard together will reinforce each other.

The sirene sounds equally when plunged in water, and fed by a current of that fluid, as in air; thus proving that it is the number of impulses alone, and nothing depending on the nature of the medium in which the sound is excited, that influences our appreciation of its pitch.

52. In general, whatever cause produces a succession of equidistant impulses on the ear, causes the sensation of a musical sound, whether such periodicity be a consequence of periodical motions in the origin

of sound, or of the mode in which a single impulse is multiplied in its conveyance to the ear.

a. For example, a series of broad palisades set edgeways in a line directed from the ear, and equidistant from each other, will reflect the sound of a blow struck at the end of the line nearest the auditor, producing a succession of echos, which, (by reason of the equidistance of the palisades,) will reach his ear at equal intervals of time $\left(= 2 \times \frac{\text{distance of palisades}}{\text{velocity of sound}}\right)$, and will therefore produce the effect of a number of single impulses originating in one point. Thus a musical note will be heard whose pitch corresponds to a number of vibrations per second, equal to the quotient of the velocity of sound by twice the distance of the palisades.

b. A similar account may be given of the singing sound of a bullet, or other missile, traversing the air with great rapidity. The bullet being in a state of rapid rotation, and not exactly alike in all its parts, presents, periodically, at equal intervals of time and space, some protuberance or roughness first to one side, then to the other. Thus an interruption to the uniformity of its mode of cutting through the air is periodically produced, and reaches the ear in longer or shorter equal intervals of time, according as the rectilinear velocity of the bullet bears a greater or less ratio to the velocity of its rotation about its axis.

c. The echos in a narrow passage, or apartment, of regular figure, being regularly repeated at equal very small intervals, always impress the ear with a musical note; and this is, no doubt, one of the means which blind persons have of judging of the size and shape of any room they happen to be in.

d. But the most ordinary ways, in which musical sounds are excited and maintained, consist in setting in vibration elastic bodies, whether flexible, as stretched strings, or membranes; or rigid, as steel springs, bells, glasses, &c.; or columns of air of determinate length enclosed in pipes. All such vibrations consist in a regular, alternate motion to and fro of the particles of the vibrating body, and are

performed in strictly equal portions of time. They are, therefore, adapted to produce musical sounds, by communicating that regularly periodic initial impulse to the aerial molecules in contact with them which such sounds require. We shall, therefore, proceed to consider more particularly the principal of these modes of production, but especially, at present, the first and last, being the most simple cases.

CHAPTER II.

VIBRATIONS OF MUSICAL STRINGS OR CORDS.

53. If a string, or wire, be stretched between two fixed pins, or supports, and then struck, or drawn a little out of its straight line, and suddenly let go, it will vibrate to and fro, till its own rigidity, and the resistance of the air, reduce it to rest. But if a *bow*, (which is an instrument composed of a bundle of fibres of horse hair, loosely stretched, and rendered adhesive by rubbing with rosin,) be drawn across it, the vibrations are continually renewed, and may be maintained for any length of time, and a musical sound is heard corresponding to the rapidity of the vibration.

54. It is required to investigate the motions of a stretched cord.

a. The mathematical theory of the vibrations of a stretched cord is remarkable, in an historical point of view, as having given rise to the first general solution of an equation of partial differen-

ces; and led geometers to the consideration of the nature and management of the arbitrary functions which enter into the integrals of these equations.

b. Let MN (fig. 9.) be a cord maintained by any means in a constant state of equal tension throughout, and disturbed by any external cause from its rectilinear position, and then left to take its own form and motion in consequence of its tension; its gravity, however, being neglected.

Let $MABCEN$ be the figure of the cord after the lapse of any time t from the initial disturbance; respecting which we will only suppose that the distance of all its points from the axis VT, (the undisturbed rectilinear position of the cord,) is extremely small; so that in this theory, as in that of the sonorous vibrations of the air, we concern ourselves only with such excursions of the vibrating molecule as may be considered infinitely minute.

c. Let A, B, C, E, be points of the cord infinitely near each other; and erecting the ordinates AP, BQ, CR, ES, and drawing Aa, Bb, Cc, Ee, parallel to VT, put

$$VP = x, \quad VQ = x', \quad VR = x'', \text{ \&c.}$$

and

$$AP = y, \quad BQ = y', \quad CR = y'', \text{ \&c.}$$

Let the tension of the cord at rest be represented by c, which, (since the cord is infinitely little disturbed from its position of repose,) will also be its tension in its disturbed state; and will in this, as in the former state, be uniform over its whole length, the curvature being evanescent.

d. The point B of the cord will, then, be solicited towards the axis by the tension c applied at B, and acting in the direction BA, and whose resolved value is, therefore,

$$c \cdot \frac{Ba}{BA} = c \cdot \frac{Ba}{Aa} = c \cdot \frac{dy}{dx},$$

since BA only differs from Aa by infinitely small quantities of the second order, which may be neglected.

Similarly the point B will be solicited *from* the axis by the tension c applied at B in the direction BC, whose resolved part in the direction of the ordinate is equal to $c \cdot \frac{d\,y'}{d\,x'}$.

The resolved parts in directions parallel to the axis, being equal and parallel, destroy each other; consequently, the whole force applied at B will be

$$c \cdot \left(\frac{d\,y'}{d\,x'} - \frac{d\,y}{d\,x}\right) = c\,\frac{d^2\,y}{d\,x^2}\,d\,x,$$

tending to increase the value of y.

e. Now the motion of the cord will be the same, whether we regard it as a continuous mass, or compound of detached particles situated at A, B, C, E, &c. and connected by filaments AB, BC, &c. without weight. Thus at B we may conceive to be placed a weight equal to

$$\frac{AB + BC}{2} = \frac{A\,a + B\,b}{2} = \frac{d\,x + d\,x}{2} = d\,x.$$

f. The mass $d\,x$ is then to be moved by the force $c \cdot \frac{d^2\,y}{d\,x^2} \cdot d\,x$.
Hence the equations of motion are

$$\frac{d^2\,y}{d\,t^2} = 2\,g\,c \cdot \frac{d^2\,y}{d\,x^2}. \qquad \text{(XXIII.)}$$

g. This equation is precisely similar to (II.) and its integral will, of course, be of the same form, or

$$y = F\,(x + \sqrt{2\,g\,c} \cdot t) + f\,(x - \sqrt{2\,g\,c} \cdot t). \qquad \text{(XXIV.)}$$

The determination of the arbitrary functions in this equation will depend on the conditions, from which we may set out ~~from~~.

55. When the preceding analysis is applied to the case of an initial disturbance of small extent, it leads to the result, that the velocity of a pulse, or undulation along a stretched cord, is equal to that which a heavy body would acquire by falling freely through

the length of a portion of the cord whose weight is equal to its tension.

a. Let us suppose the cord to be of indefinite length, and the part initially disturbed to be *comparatively* very small; and having an indefinite undisturbed portion on either side. In this case, it is evident by the very same reasoning as that of art. 19, that a pulse, or undulation, will run out both ways along the cord from the point of initial disturbance, with a velocity represented by

$$V = \sqrt{2gc}, \qquad \text{(XXV.)}$$

every molecule of the cord being once agitated, during the time the pulse runs over it, and no more.

b. Since, in the above investigation, c represents a *force* equal to the tension on the same scale that dx represents a *weight* equal to that of the element dx, we have

$$\text{weight of } dx, : \text{tension} :: dx : c.$$

Hence c represents the length of a portion of the cord whose weight is equal to the tension; and $\sqrt{2gc}$ is the velocity which would be acquired by a body falling freely by gravity through that length.

c. The coincidence of this result with experiment has been put to careful trial by Weber, and a more complete one could not have been wished.

56. If the cord is attached at one of its extremities to an immovable point, when the undulation reaches this point, it will be reflected from it; and it is easily proved by means of the formula (XXIV.), that the reflected pulse runs back with the same velocity as the direct, and is in all respects similar and equal to it, only that it lies on the opposite side of the axis.

57. If the cord be fixed at both ends, the two pulses, into which the initial pulse has separated itself, will

each be totally reflected, and will run along the whole length, being reflected again at the other end, and thus run backwards and forwards for ever, if we neglect the effect of the stiffness of the cord and resistance of the air.

If the cord is struck, or forcibly drawn out of its situation, it will continue to vibrate to and fro; and the time of one complete vibration, after which it resumes its initial state, is equal to that of a pulse running over double the length of the cord.

a. Suppose the whole length of the cord to be

$$l + l' = L,$$

of which l lies on the positive, and l' on the negative, side of the origin of x. That portion of the subdivided primitive pulse, which runs towards the positive side of the x, will describe the length l in a time $= \frac{l}{\sqrt{2gc}}$; being then reflected, it will describe the whole length L in a time $\frac{L}{\sqrt{2gc}}$; and being again reflected, it will describe l' in a time $\frac{l'}{\sqrt{2gc}}$, so that after a time

$$\frac{l}{\sqrt{2gc}} + \frac{L}{\sqrt{2gc}} + \frac{l'}{\sqrt{2gc}} = \frac{2L}{\sqrt{2gc}},$$

it will reach its first starting point; and having been twice inverted by reflection, will lie now on the same side of the axis it originally was.

Similarly, the negative portion of the original pulse will describe l', L, and l, and reach its starting point after two reflections in the time

$$= \frac{l'}{\sqrt{2gc}} + \frac{L}{\sqrt{2gc}} + \frac{l}{\sqrt{2gc}} = \frac{2L}{\sqrt{2gc}},$$

the same as the other, and will also have recovered its original situation with respect to the axis. Thus at the end of this time the two pulses will precisely reunite, and constitute a compound pulse in all respects similar to the initial impulse. The state of the cord, then,

after the lapse of the time $\frac{2L}{\sqrt{2gc}}$, will, (abstracting the effects of resistance, &c.) be precisely what it was at first; and so again, after the lapse of time $\frac{4L}{\sqrt{2gc}}$, $\frac{6L}{\sqrt{2gc}}$, &c. the same state will recur, so that if left to itself it will continue to vibrate for ever.

Thus we see that what in an indefinite cord was merely a pulse running along it and never returning, becomes, by the reaction of the fixed extremities of a finite one, a regular vibration, in which each molecule repeats its motion to and fro, on either side of the axis, at equal intervals, for ever.

b. In the foregoing remarks no particular assumption has been made respecting the extent of the pulse. It has not been supposed small with respect to l, l'; and, consequently, the above conclusion applies equally to the case, where the initial disturbance is confined to a minute portion of the cord, and where a large portion, or even its whole length, is disturbed at once. Only in the former case the motions of the individual molecules of the cord will be performed by starts, interrupted by intervals of absolute rest in the axis. In the latter there will be no moments of rest but those, when the direction of the motion changes at the extreme points of their excursions.

The time of vibration of a stretched cord, whose length is L, is, then, equal to $\frac{2L}{\sqrt{2gc}}$, being the time of a pulse running over double the length of the cord.

58. Hence, the times of the vibrations of different cords are proportional to their lengths divided by the square root of their tensions; and the number of vibrations, in a given time, are proportional to the quotients of the square root of their tensions divided by their lengths.

Division of a vibrating Cord into Ventral Segments.

59. A cord, although vibrating freely, may yet have any number of points, equally distributed at aliquot parts of its whole length, which never leave the axis, and between which the vibrating portions are equal and similar, and lie alternately above and below the axis, and in reversed positions as to right and left. Such points of rest are called *nodes* or *nodal points ;* the intermediate portions which vibrate are termed *bellies* or *ventral segments.*

a. A very simple consideration will show that such may be the case; for, if we conceive two equal and similar cords, AMB, $BM'C$, (fig. 10.) both attached to the same point, B, and vibrating simultaneously, the strain on B, from both their tensions, will be always equal and opposite, provided the curves be so related as above descrbed, and B, therefore, will be retained in equilibrio, independently of its attachment to any extraneous body. Were it, therefore, detached, or were the two cords, instead of being fixed to one immovable point, to be merely linked together at B, so as to form one cord of double the length, their vibrations would be the same. And this reasoning might be extended to another point C.

b. Hence, if L be the whole length of a cord, n the number of ventral segments into which it divides itself, and, therefore, $n - 1$ the number of its nodes, the length of each ventral segment will be $\frac{L}{n}$; the time of the vibration of the cord, being the same as that of one of its ventral segments, will be $\frac{2L}{n\sqrt{2gc}}$; and the number of vibrations per second will be represented by the reciprocal of this fraction.

60. If a string in the act of vibration be touched in any point, so as to reduce that point to rest and retain it in the axis, then the string, after the contact, will

either not vibrate at all; or it will divide itself into a certain number of ventral parts, similar and equal to each other and separated by nodes; and each of these will vibrate, as if the others had no existence, but instead the nodes were fixed points of attachment.

a. For if the divisions on each side of the point, which we have supposed at rest, were not symmetrical and on opposite sides of the axis, they would exert an unequal force upon the point, and it would not be retained at rest. It must, also, be the same with the divisions on each side of the other nodes.

b. Experience confirms this. If the string of a violin, or violoncello, while maintained in vibration by the action of the bow, be lightly touched with the finger, or a feather exactly in the middle, or at one third of the length, it will not cease to vibrate, but its vibrations will be diminished in extent and increased in frequency, and a note will become audible, fainter but much more acute than the original, or, as it is termed, the *fundamental* note of the string, and corresponding in the former case to a double, in the latter to a triple rapidity of vibration. The note heard in the former case is the octave, in the latter, the twelfth above the fundamental note.

If a small piece of light paper, cut into the form of an inverted *V*, be set astride on the string, it will be violently agitated, and, probably, thrown off when placed in the middle of a ventral segment, while at a node it will ride quietly as if the string were (as it really is at those points) at perfect rest. The sounds thus produced are termed *harmonics*.

61. Any number of the different modes of vibration, of which a cord is susceptible, may be going on *simultaneously*, or be, as it were, superposed on each other.

a. This is a consequence of the principle in mechanics of "the superposition of small motions," which, when the excursions of the parts of a system from their places of rest are infinitely small, admits of any or all the motions of which, from any causes, they are susceptible, to go on at once, without interfering with or disturbing each other.

Coexistence of several Modes of Vibration in one Cord.

b. In the particular case before us it is easily shown; for, since the general integral of the equation (XXIII.)

$$\frac{d^2 y}{d t^2} = 2 g c \cdot \frac{d^2 y}{d x^2},$$

is (XXIV.)

$$y = F(x + \sqrt{2gc} \cdot t) + f(x - \sqrt{2gc} \cdot t),$$

where F and f denote arbitrary functions, we may suppose

$$F(x) = F'(x) + F''(x) + F'''(x) +, \&c.$$
$$f(x) = f'(x) + f''(x) + f'''(x) +, \&c.$$

where F', F'' &c., and f', f'', &c., denote functions equally arbitrary, and we get

$$y = [F'(x + \sqrt{2gc} \cdot t) + f'(x - \sqrt{2gc} \cdot t)]$$
$$+ [F''(x + \sqrt{2gc} \cdot t) + f''(x - \sqrt{2gc} \cdot t)] + \&c.,$$

Now each of the expressions within brackets is the integral of an equation exactly similar to the original one. Therefore if we put

$$\frac{d^2 y'}{d t^2} = 2 g c \frac{d^2 y'}{dx^2},$$
$$\frac{d^2 y''}{d t^2} = 2 g c \frac{d^2 y''}{dx^2}, \&c.$$

we shall have

$$y = y' + y'' + y''' +, \&c.$$

Thus, if the several particular modes of vibration,

$$y = y', \quad y = y'', \&c.,$$

be possible,

$$y = y' + y'' +, \&c.$$

will also be possible.

c. The ordinate of the curve, into which the cord at any moment forms itself in virtue of the compound vibration, will be the sum (algebraically understood) of the ordinates it would have in virtue of each simple one separately. The compound curve will be formed by first constructing on the abscissa, as an axis, any one of the simple ones; then, on that curve, as an abscissa, any other; on the new curve then arising any other; and so on.

Hence it is evident that if we suppose the curve, whose ordinate is y', to be of the form, fig. 11, (a), having no node; and that, whose ordinate is y'', to have, for instance, one node, as fig. 11, (b); the corresponding modes of vibration, when coexisting, will produce a curve, such as (c). On these we may superpose a third mode of vibration, where the string divides itself into three ventral segments, as (d), and the result will be a curve, such as (e), and so on to any extent. The reader may exercise himself in tracing the variations of form in these curves, as they go through the several phases of their periodic excursions during one complete period of a vibration of the whole string as one cord.

d. Experience again confirms this result of theory. It was long known to musicians that, besides the principal or fundamental note of a string, an experienced ear could detect in its sound, when set in vibration, especially when very lightly touched in certain points, other notes, related to the fundamental one by fixed laws of harmony, and which are called, therefore, harmonic sounds. They are the very same, which, by the production of distinct nodes, may be insulated as it were, and cleared from the confusing effect of coexistent sounds, as in art. 60. They are, however, much more distinct in bells, and other sounding bodies, than in strings, in which only delicate ears can detect them.

e. The production of harmonic sounds from cords, and their division into aliquot parts, was first noticed in 1677 by Wallis, but the subject remained unattended to, till taken up by Sauveur in a valuable memoir, which first put this part of the doctrine of acoustics in a clear point of view.

62. The monochord is a very simple instrument, well adapted to exhibit these and all other phenomena of vibrating strings.

a. It is nothing more than a single string of catgut, fixed at one end immovably, and at the other strained over a well-defined edge, which effectually terminates its vibrations, either by a known weight

or by screws. A similar well-defined edge is also interposed between its fixed end and the vibrating portion, and the interval between the two edges is graduated into aliquot parts, or in any other convenient way; and it is provided with a movable *bridge*, or piece of wood capable of being placed at any division of the scale, and abutting firmly against the string so as to stop its vibrations, and divide it into two of equal or unequal lengths, as the case may be.

b. By the aid of this instrument we may ascertain the number of vibrations which belong to any assigned musical note, or which correspond to the notes of any musical instrument, as a piano-forte, &c. For when we have ascertained the weight of a known length of the catgut, of which the string is formed, and the weight which must be applied to stretch the cord, so as to make its fundamental tone coincide with any given note (as the middle *G* of a piano-forte); then by the formula $\frac{\sqrt{2gc}}{2L}$ we know the number of complete vibrations going and returning, and by the formula $\frac{\sqrt{2gc}}{L}$ the number of oscillations from rest on one side of the axis to rest on the other, that is, the number of impulses made on the ear per second corresponding to that fundamental tone.

To determine the same for any note *sharper*, *higher*, or more *acute* than the fundamental note, we have only to apply the bridge, and move it backwards and forwards, till the sound of the vibrating part of the string is *in unison with* that of the note to be compared, of which the ear judges with the greatest precision; then, if the length of this part, read off on the divided scale, be called l; the number of its vibrations per second will be to that of the whole string $L :: L : l$, and is therefore known.

63. The contact of a solid body is not the only way of producing nodal points. If two cords equally tended, and in all other respects similar, but one only half, one third, or other aliquot part of the length of the

other, be placed side by side, and the shorter be struck or sounded, the vibration will be communicated to the longer by the intervention of the air. The longer cord will thus at once be thrown into a mode of vibration, in which the whole length is divided into ventral segments, each equal to the shorter string.

To understand how this may happen, let us conceive first two strings of equal length, one at rest, the other vibrating; and let them be placed parallel, and side by side; then the sonorous pulses, diverging at any instant from each point of the moving string, will arrive at once at each corresponding point of the other. The aerial molecules in their progress, while condensed, will press on the string and give it a very slight motion in their own direction; in their retreat they will be followed by the string, whose vibrations by hypothesis are synchronous with their own; but it will not follow them so fast as they retreat, and it will therefore be urged and accelerated by those behind. It will, however, come to rest, in its furthest point of excursion, at the same time with the aerial molecules, when its elasticity will begin to urge and accelerate it in the contrary direction. But now also the direction of the motion of the air has changed, and again conspiring with that of the cord still continues to accelerate it, and so on, till, after a very great number of repetitions of this process, the cord will be set in full vibration and will become itself a source of sound.

But its sound will always be much fainter than that of the original vibrating cord, for this reason, *viz.* that its acquired motion is perpetually dissipated, *laterally*, into the surrounding air; for no cord is so exactly uniform, or so equally tended in every part of its transverse section, that it *can* vibrate rigorously on one plane. Hence it will inevitably begin to rotate, or to describe vibrations, whose plane is constantly shifting, and thus it will throw off laterally a great part of the motion it receives from the air; just as a body exposed to the radiation of a hot fire never acquires a temperature equal to that of the fire, part of the heat communicated being dissipated by lateral radiation.

Sympathetic Communication of Vibrations.

64. Just as a small pull, repeated exactly in the time of its natural swing, will raise a great bell, or a trifling impulse a heavy pendulum, so the molecules of the air, in a state of sonorous vibration, will impress on any body, *capable of vibrating in their own time*, an actual vibratory motion; and if a body be susceptible of a number of modes of vibration performed in different times, that mode only will be excited which is synchronous with the aerial pulsations. All other motions, though they may be excited for a moment by one pulsation, will be extinguished by a subsequent one.

In the vibrations of cords, which from their small surface can receive but a trifling impulse from the air, the sounds and motions excited by this sort of sympathetic communication are feeble; but in vibrating bodies, which present a large surface, they become very great.

It is a pretty well authenticated feat, performed by persons of a clear and powerful voice, to break a drinking-glass by singing its proper fundamental note close to it. Looking-glasses also are said to have been occasionally broken by music, the excursions of their molecules in the vibrations, into which they are thrown, being so great as to strain them beyond the limits of their cohesion.

65. The way, in which the permanent or vibratory oscillations of a cord arise by reflection at its fixed extremities from a wave propagated along it progressively, may be rendered a matter of ocular inspection, if we take a long and pretty thick cord, fasten it at one end, and holding the other in our hands, give it a regular motion to and fro, transverse to the length of the cord.

Progressive waves will thus arise, which, as soon as they reach the fixed end, and are reflected, will be observed to interfere with those still on their way, and, as it were, to arrest them, producing a series of nodes and ventral segments, whose number will depend on the tension and frequency of the alternate motion communicated to the movable end.

In this arrangement the continual periodic renewal of the primary impulse by the hand supplies the place of a reflecting obstacle at that end.

66. The pitch of the sound of a vibrating string depends only on the number of vibrations made in a given time ; but its quality will depend partly on the nature of the string, and especially on its equality of thickness; besides which, much may depend on the form and extent of the wave excited, or of the curve into which it is thrown.

In instruments, like the violin or violoncello, played with a bow, or the guitar or harp, where the string is drawn softly out of its position and suddenly let go, this curve is probably single, and occupies the whole extent of the string. But in the piano-forte, where the strings are struck, near one extremity, with a sharp, sudden blow, there can be little doubt that the vibration consists in an elevation or bulge, more or less extensive, running backwards and forwards. Fig. 12. represents the different phases of a single complete vibration of a string so struck. The first wave (*a*) is a single elevation; it divides in (*b*) into two, running contrary ways; in (*c*) that nearest the end *A* is reflected and takes a reversed position; in (*d*) they advance the same way towards *B*; in (*e*) the unreflected portion reaches *B*, is there reflected and reversed as in (*f*). In (*g*) it meets and coincides with the former reflected portion, there forming a depression equal and similar to the original elevation in (*a*), and as far distant from the end *B* as the former from *A*. After this the same steps are repeated in the reverse direction, till the original elevation is reproduced again as in (*a*). The waves,

however, must be supposed to bear a much more considerable ratio to the whole string than in the figure. It is evident that the magnitude of this ratio must influence the quality of the tone, and thus a difference of character in the tone, according as the keys are struck with quick, short, brilliant blows; or gently pressed, and the duration of the contact of the hammers with the strings prolonged for an instant of time, giving rise to a more moderate but sustained *tenuto* effect, by bringing a larger portion of the string, or even the whole, into motion at once.

But whether the portion disturbed at once be large or small, whether it occupy the whole string, or run along it like a bulge in its line; whether it be a single curve, or composed of several ventral segments with intervening nodes; we must never lose sight of the fact, that the motion of a string with fixed ends is no other than an undulation or pulse continually *doubled back on itself*, and retained constantly within the limits of the cord, instead of running out both ways to infinity.

67. It very seldom can happen that the vibrations of a string actually lie in one plane. Most commonly they consist of rotations more or less complicated, except when produced by the sawing of a bow across the string, when they are forcibly limited to the plane of motion of the bow.

The real form of the orbit, described by any molecule, may be made matter of ocular inspection, by letting the sun shine through a narrow slit, so as to form a thin sheet of light. Let a polished wire be placed so as to penetrate this sheet perpendicularly to its plane, and the point where it cuts the plane will, at rest, be seen as a bright speck, but when set vibrating, it will form a continued luminous orbit, just as a live coal whirled round appears as a circle of fire. Fig. 13. exhibits specimens of such orbits, observed by Dr. Young.

68. A very curious case of a mode of vibrations, by which a string may be made to produce a sound graver than its fundamental note, is mentioned by M. Biot.

If an obstacle be placed below the middle point of a vibrating string, so as just to touch, but not to press against it, and the string, be then drawn up vertically and let go, it will strike at every oscillation upon this obstacle, and bend over it, as in fig. 14., at every blow; thus resolving itself into two of half the length. Thus the first semi-oscillation will be performed as a whole, the next as a subdivided string.

Let unity represent the time of a complete oscillation from rest to rest of the whole string; then will the times, in which the different phases of the motion now in question are performed, be as follows:

From the position ABC to the straight line AC	$= \frac{1}{2}$
From the position AC to the position AED, DFC	$= \frac{1}{4}$
Back to the straight line	$= \frac{1}{4}$
Back to the original position ABC . . .	$= \frac{1}{2}$
Sum	$= \frac{3}{2}$

Thus the interval between two consecutive blows made by the string on the bridge is $\frac{3}{2}$ of the time of oscillation of the string as a whole, from rest on one side of the axis to rest on the other, or of the impulses made by it on the ear when so vibrating. Hence, the blows on the bridge will be heard as a continued note, (though extremely harsh and disagreeable,) graver than that of the string vibrating as a whole, by the musical interval called a fifth.

CHAPTER III.

VIBRATIONS OF A COLUMN OF AIR OF DEFINITE LENGTH.

69. The general equation representing the motions of the molecules of a stretched cord of indefinite length is, as we have seen, precisely similar in its form, and in that of its complete integral, to that of the particles of air in a sounding column. There subsists, of course, a perfect analogy between the two cases; and, *mutatis mutandis*, all propositions, which are true of a vibrating cord, are also true of a vibrating cylindrical column of air.

70. Thus, if such a cylindrical column be inclosed in a pipe, stopped at both ends by perfectly immovable stoppers, and if we suppose any single impulse communicated to one of its sections, this will immediately divide itself into two pulses running opposite ways; they will be totally reflected at the two extremities, will pass over the whole length of the pipe, and be reflected again at the opposite extremities, and so on, crossing each other at each traverse. This motion will be continued, till it is destroyed by friction and by the imperfect fixity and rigidity of the stoppers, which allow some of it to pass into them and be lost at each reflection.

If the length of the pipe is $L = l + l'$, and l is the distance of the impulse from one of the extremities (A); one of the pulses,

after describing the space l before and l' after reflection, will meet the other, which has described l' before and l after reflection, at a distance $= l$ from the other extremity (B), and produce a compound agitation in the section at that place similar to the primitive disturbance; thence the partial pulses will again diverge, and after undergoing another reflection will again unite in their original point of departure, constituting a repetition of the first impulse.

71. If the section first set in motion be maintained in a state of vibration, synchronous with the return of the reflected pulse, it will unite with and reinforce it at every return; and the result will be a clear and strong musical sound, resulting from the exact combination of the original periodic impulse with all its echos. This will be transmitted through the pipe to the outer air, and thus dissipated and lost.

For simplicity, let us suppose the section primitively set in vibration, and so maintained, to be situated just in the middle of the pipe. Then, when once the regular periodic pulsation of the contained air is established, it is evident that the motion of the column will consist of a constant and regular fluctuation to and fro within the pipe, of the whole mass, the air being always condensed in one half of the pipe while it is rarefied in the other.

The greatest excursions from their place will be made by the molecules in the middle, while those at the extremities, being constantly abutted against the stoppers, remain unmoved, and the excursions made by each intermediate molecule will be greater the nearer it is to the middle.

On the other hand, the rarefactions and condensations are greatest at the extremities, and diminish as we approach the middle of the pipe, where there is neither condensation nor rarefaction.

72. In the same way as a vibrating cord is susceptible of division into its several aliquot parts all vibrat-

ing simultaneously, so may the aerial column in our stopped pipe vibrate in distinct ventral segments.

The manner in which this may take place will be evident on inspection of figs. 15. and 16., where the arrows denote the directions of the motions of the vibrating molecules, and where we see the immobility of the nodal sections is secured by the equal and opposite pressures of the molecules on either side of them. At these nodal sections, too, the same thing holds good as at the stopped extremities, their molecules remain constantly at rest, while yet they undergo greater vicissitudes of compression and dilatation than those in any other parts of the column.

73. Precisely, too, as in the vibrations of strings, any number of these modes of vibration may go on simultaneously.

Such combined modes may be produced by an expert flute player, by a nice adjustment of the force of his breath; at least the octave of any note may be obtained without difficulty, and distinctly heard with the fundamental note.

74. If, at a point half way between two nodes, (regarding the stopped ends as nodes,) we conceive a narrow ring of the cylindrical pipe, in which the vibrating column is contained, to be cut away, so as to open a free communication with the outer air; it will no way alter the vibrations of the column; nor will the opening of a hole in the pipe at this place affect them. Also the sound of the pipe may be excited and maintained by placing at this hole a vibrating body, whose vibrations are executed in equal times with those in which the excursions to an fro of the included sections are performed in the stopped pipe. Such an aperture is called an *embouchure*.

a, For, at such a point, the condensations and rarefactions are evanescent, and the amplitudes of the molecular excursions are at a maximum; consequently, there will be no tendency for the air to pass in or out; neither will its motion be impeded, being parallel to the axis of the column and without any lateral bias.

b. But let us now conceive the one half (A) of the pipe entirely removed, and in its place a disc substituted exactly closing the aperture, and maintained, by some external cause, constantly in a state of vibration, such, that the performance of one complete vibration, going and returning, shall exactly occupy as much time as a sonorous pulse would take to traverse the whole length of the stopped pipe ($A + B$), or double that of the open one (B).

Its first impulse on the air will be propagated along the pipe (B) and reflected at the stopped end, and will again reach the disc just at the moment when the latter is commencing its second impulse. But, the absolute velocity of the disc in its vibrations being excessively minute compared with that of sound, the reflected pulse will undergo a second reflection at the disc, as if it were a fixed stopper. It will, therefore, in its return exactly coincide and conspire with the second original impulse of the disc; and the same process being repeated on every impulse, each will be combined with all its echos, and a musical tone will be drawn forth from the pipe, vastly superior to that which the disc vibrating alone in free air would produce. This is, in fact, the simplest instance of the resonance of a cavity; of which more hereafter.

75. The fundamental sound of a pipe open at one end is the same with that of a pipe closed at both ends, and of double the length.

For, in the preceding example, it is manifestly of no importance, whether the pulses reflected from the closed end of the pipe (B) undergo a second reflection at the disc, and are so returned back by the pipe, or whether we regard the disc as penetrable by the pulse, (i. e. a mere imaginary vibrating section,) and suppose the pulse to run on and be reflected at the extremity of the other half

(A) of the bisected pipe ($A + B$), and on its return again to pass freely through the disc and be again reflected at the stopped extremity of (B). The sounds produced will be the same, on the principle of the superposition of vibrations.

76. The vibrations of a pipe open at one end may be easily excited and maintained by means of a vibratory body placed before it and a strong sound produced.

Let any one take a common tuning-fork, and on one of its branches fasten with sealing-wax a circular disc of card of the size of a small wafer, or sufficient nearly to cover the aperture of a pipe. The sliding joint of the upper end of a flute, with the mouth-hole stopped, is very fit for the purpose, it may be tuned in unison with the loaded tuning-fork (a *C* fork) by means of the movable stopper, or the fork may be loaded till the unison is perfect.

If the fork be then set in vibration by a blow on the unloaded branch, and the disc be held close over the mouth of the pipe, as in fig. 17., a note of surprising clearness and strength will be heard. Indeed, a flute may be made to *speak* perfectly well by holding close to the embouchure a vibrating tuning-fork, while the fingering proper to the note of the fork is at the same time performed.

77. The most usual means of exciting the vibrations of a column of air in a pipe is, by blowing into, or rather over it, either at its open end or at an orifice made for the purpose at the side; or by introducing a small current of air into it through an aperture of a peculiar construction called a *reed.*

a. The reed is provided with a *tongue*, or flexible elastic plate, which nearly stops the aperture, and which is alternately forced away by the current of air and returns by its elasticity, thus producing a continued and regularly periodic series of interruptions to

the uniformity of the stream, and, of course, a sound in the pipe corresponding to their frequency. Except, however, the reed be so constructed as to be capable of vibrating in unison, or nearly so, with, at least, one of the modes of vibration of the column of air in the pipe, the sound of the reed only will be heard, the resonance of the pipe will not be called into play, and the pipe will not *speak;* or will speak but feebly and imperfectly, and yield a false tone.

b. But of reeds more hereafter; at present, let us consider what takes place, when the vibrations of a column of air are excited by blowing over the open end of a pipe, or an aperture in its side. To do it effectually, the air must be directed in a small current, not *into*, but *across* the aperture, as in fig. 18., so as to graze the opposite edge. By this means a small portion will be caught and turned aside down the pipe, thus giving a first impulse to the contained air, and propagating down it a pulse in which the air is slightly condensed. This will be reflected at the end as an echo, and return to the aperture where the condensation goes off, the section condensed expanding into the free atmosphere. But in so doing, it lifts up, as it were, and for a moment diverts from its course the impinging current, and thus, while it passes, suspends its impulse on the edge of the aperture. The moment it has escaped, the current resumes its former course, again touches the edge of the aperture, creates there a condensation, and propagates downwards another condensed pulse, and so on.

Thus the current passing over the aperture is kept in a constant state of *fluttering* agitation, alternately grazing and passing free of its edge, at regular intervals, equal to those in which a sonorous pulse can run over twice the length of the pipe; or, more generally, in which the condensations and rarefactions recur at its aperture in virtue of any of the modes of vibration of which the column of air in the pipe is susceptible.

78. Wherever there is a communication opened between the column of air in a pipe and the free atmo-

sphere, that point will become a point of maximum excursion of the vibrating molecules, or the middle of a ventral segment.

In such a point the rarefactions and condensations vanish, the air reducing itself constantly to an equilibrium of pressure with the free atmosphere with which it is in contact. Hence, if the pipe speak at all, it will take such a mode of vibration as to satisfy this condition, but, consistently with this, it may divide itself into any number of ventral segments.

79. A pipe open at both ends can yield, if properly excited, a musical sound.

The column of air in it vibrates in the mode represented in fig. 19., where there is a node in the middle, and each ventral segment is only half a complete one.

80. It is required to represent, in an algebraic formula, the time of vibration of a column of air, or the number of vibrations per second, corresponding to any mode of vibration.

a. If the pipe is closed at both ends, as in fig. 16., let the number of nodes, including the two stopped ends, be n, length of the pipe L, and V the velocity of sound in feet per second.

The number of segments into which the pipe is divided will be $n - 1$; and the length of each segment will be $\frac{L}{n-1}$.

This length is travelled by a sonorous pulse in a time, equal to the length divided by the velocity of sound

$$= \frac{1}{n-1} \cdot \frac{L}{V},$$

and this is the time of vibration of the middle section of it to which the sound corresponds.

The number of vibrations per second is, therefore,

$$\frac{\text{one second}}{\text{time of vibration}} = (n - 1) \cdot \frac{V}{L},$$

in which any integer may be substituted for n greater than unity.

Making, therefore, $n = 2, 3, 4$, &c. successively, we obtain the following series of the numbers of vibrations, corresponding to the different tones of the pipe;

$$1 \cdot \frac{V}{L}; \quad 2 \cdot \frac{V}{L}; \quad 3 \cdot \frac{V}{L}, \text{ \&c.}$$

b. If the pipe is open at one end, as in fig. 20., the stopped end must be regarded as a node. Calling the whole number of nodes thus included n, the number of complete ventral segments will be $n - 1$, and one half segment will terminate at the open end. Therefore $2(n - 1) + 1 = 2n - 1$, will be the number of such halves contained in the length L.

The length of each complete segment is, therefore, $\frac{2L}{2n-1}$; and the number of vibrations per second is

$$\frac{2n - 1}{2} \cdot \frac{V}{L},$$

in which any integer may be substituted for n.

Making, therefore, $n = 1, 2, 3$, &c., we obtain the following series of the number of vibrations, corresponding to the different tones of the pipe,

$$\tfrac{1}{2} \cdot \frac{V}{L}; \quad \tfrac{3}{2} \cdot \frac{V}{L}; \quad \tfrac{5}{2} \cdot \frac{V}{L}, \text{ \&c.}$$

c. If the pipe is open at both ends, as in fig. 21., let the number of nodes be n. The number of complete ventral segments will be $n - 1$, and there will be a half one at each end, making n segment in the length L.

The length of each complete segment is, therefore, $\frac{L}{n}$; and the number of vibrations per second is

$$n \cdot \frac{V}{L},$$

in which any integer may be substituted for n.

Making, therefore, $n = 1, 2, 3$, &c., we obtain the following series of the number of vibrations, corresponding to the different tones of the pipe,

$$1 \cdot \frac{V}{L};\ 2 \cdot \frac{V}{L};\ 3 \cdot \frac{V}{L},\ \&c.$$

d. Taking, therefore, for unity the number of vibrations per second in the fundamental tone, the series of harmonics will run as follows;

In a pipe stopped at both ends . . .	1, 2, 3, 4, 5, &c.
——— open at both ends . . .	1, 2, 3, 4, 5, &c.
——— open at only one end . . .	1, 3, 5, 7, 9, &c.

It being recollected, however, that in the last series the fundamental note 1 is an octave lower than in the others, that is, performs its vibrations only half as rapidly.

81. To produce the different sounds of which a pipe is susceptible, it is only requisite to begin with as gentle a blast as will make the pipe speak, and to augment its force gradually.

a. The fundamental tone will be heard first; and as the strength of the blast increases, will grow louder, till at length the tone all at once starts up an octave, that is, the interval between notes whose vibrations are as 1 : 2.

By blowing still harder, the next harmonic 1 : 3, or, as it is called in music, the octave of the fifth, or the *twelfth* of the fundamental tone, is heard; but no adaptation of the embouchure, or force of the wind, will produce any note intermediate between these.

The next harmonic is 1 : 4, and corresponds to the double octave, or *fifteenth* of the fundamental tone; and the next or 1 : 5, to the *seventeenth*, or major third above the double octave. The next, 1 : 6, corresponds to the *nineteenth*, or double octave of the fifth, and so on. (The explanation of these terms will be given hereafter).

All the notes here enumerated are very readily produced on the flute, without changing the fingering, from the lower *C* or *D* upwards, by merely varying the force of the blast, and a little humoring the form of the lips and their position with respect to the embouchure.

M. Biot, by adapting an organ bellows to regulate the blast, and give it the requisite force and uniformity, succeeded in drawing from a pipe furnished with a proper embouchure, not only these, but also the notes, represented in the harmonic series by 7, 8, 11, and 12, but not 9 and 10, (the reason of which vacancy does not appear.)

b. The rationale of the continual subdivision of the vibrating column, as the force of the blast increases, is very obvious. A quick sharp current of air, is not so easily driven aside by an external disturbing force; and, when so driven, returns more rapidly to its original course, than a slow and feeble one. A quick stream, when thrown into a ripple by an obstacle, undulates more rapidly than a slow one. Consequently, on increasing the force of the blast, a period will arrive when the current *cannot* be diverted from its course and return to it so *slowly*, as is required for the production of the fundamental note. The next higher harmonic will then be excited, until, the force of the blast increasing, it becomes once more incapable of sympathizing with the excursions of the aerial molecules at the embouchure in this mode of vibration, and so on.

82. If we know the velocity of sound in the column of air included in a pipe, the length of the pipe, and the mode of vibration, the number of vibrations may be computed; and *vice versâ*, if we know the number of vibrations made in a given pipe, vibrating in a known manner, we may thence compute the velocity of sound. This furnishes a ready and simple method of determining the velocity of sound in any gas or vapor.

We have only to fill a pipe first with air, and then with the gas or vapor in question, and, having set them vibrating by any proper means, so as to draw forth their fundamental tone, to compare this with a monochord, or with any musical instrument possessing a regular scale or progression of notes, whose vibrations are known; and having thus ascertained the number of vibrations per second, performed by a column of each medium, the velocities of sound in the respective cases will be in the direct ratio of their numbers.

It is thus that Chladni, and more lately, Van Rees, Frameyer, and Moll have ascertained the velocities of sound in the various media enumerated in art. 24.

83. That it is really the air which is the sounding body in a flute, organ-pipe, or other wind instrument, appears from the fact, that the materials, thickness, or other peculiarities of the pipe, are of no consequence.

A pipe of paper and one of lead, glass, or wood, provided the dimensions be the same, produce, under similar circumstances, exactly the same tone as to *pitch*. If further proof were necessary, the difference of pitch produced by filling the pipe with different gases would place the point beyond a doubt.

84. The difference in the *qualities* of the tones, produced by different pipes, is to be attributed to the friction of the air within them, setting in feeble vibration their own proper materials.

85. The influence of the size and situation of the embouchure of a pipe, and still more of the manner of exciting the vibrations of the sections of the aerial column near that place, are very material in determining the pitch of the tone uttered.

a. Were it possible to excite the aerial column to vibration by setting in motion a single section of it by a wish, we should obtain,

doubtless, sounds always strictly conformable to the length of the pipe and its harmonic subdivisions as above; but, in fact, the vibrating column of air and the extraneous body, (be it reed, tuning-fork, or stream of air,) which sets it in motion, exercise on each other a mutual influence; they vibrate as a system, and the resulting tone may be made to deviate more or less from the pure fundamental tone of the pipe, according to the greater or less mass of matter, and the fixity of the vibrations of the apparatus by which the pipe is made to *speak*.

b. When, for example, the cause of vibration is the mere passage of a stream of air over the orifice, whose motions are almost entirely commanded by the condensations and rarefactions within the pipe, but little deviation can take place. Yet, by varying the inclination of the stream, (as in the case of the flute by turning the mouth-hole more inwards or outwards with respect to the lips,) and thus giving it a greater or less obliquity to the edge against which it strikes, we may alter the note very sensibly, as is known to all flute players. They, indeed, use this means of humoring the instrument, and playing in tune in keys which would otherwise be insupportable.

86. In the diapason organ-pipe, whether open or stopped, a stream of air is admitted at the vertex of the conical lower end, but is prevented from passing through the whole length of the pipe by a plate of metal, separating the cone from the pipe, and is forced to escape through a narrow slit transverse to the axis of the pipe, in doing which it strikes against the edge of a thin piece of lead, or other flexible metal.

This disposition will be understood by the inspection of the fig. 22., in which *BB* is the organ-pipe, and *b c b* the conical appendage at its foot by which the air is admitted. One side of the pipe *BLM* is flattened and a little bent inwards, and at *L* a narrow slit is made, just opposite to the lower edge of which is the plate

of metal *b b*, which has its edge nearest the orifice a little cut away, so as not quite to fill the whole section of the pipe, but to leave a narrow slit parallel to the slit *FF* in the side of the pipe. Through this the air admitted at *c* escapes, and is directed in a thin sheet against the upper lip *L* of the lateral slit; against which it breaks, as described in art. 77, and sets in vibration the column of air contained in the pipe.

87. Harmonics of the fundamental note of the diapason pipe may be produced, either by increasing the force of the stream of air, or by diminishing the breadth of the slit through which the air escapes.

a. If the stream of air be increased in strength, the pipe will yield the octave and harmonics of its fundamental note, forming the series 1, 2, 3, 4, &c.

b. If, on the other hand, the current of air remaining constant, the breadth of the slit through which the air escapes be diminished, according to the experiments of Messrs. Biot and Hamel, harmonics will also be produced, but in the progression 1, 3, 5, 7, &c., the octaves of the fundamental note and of all the others being entirely wanting.

88. In reed-pipes, or those in which the vibrations are excited and maintained by passing a current of air into a pipe through a reed, the influence of the reed on the pipe is very great. The most perfect and pure tone is produced, of course, when the reed and the pipe separately are pitched in unison, but a considerable latitude in this respect exists; and within certain limits, depending on the mass and stiffness of the reed, as compared with the dimensions of the pipe, a power of mutual accommodation subsists, and a mean tone is produced, less powerful, and less pure and pleasing,

however, as the pipe is more forced from its natural pitch, until it ceases to sound altogether, and the note produced, if any, is that of the reed alone.

a. In this respect there is, however, a great difference in pipes of different sizes. In large organ-pipes the reed vibrates with nearly the same freedom as in the open air, and will, therefore, *speak* when the pipe has ceased to resound. But in small and narrow pipes, as in oboes and other similar wind-instruments, a much closer correspondence between the pitches of the reed and pipe is required, or the reed will not vibrate.

b. Messrs. Biot and Hamel adapted to a glass pipe a reed of the ordinary construction, represented in fig. 23, in which the vibrating tongue *L*, (by whose oscillations the opening of the reed at *R* is alternately opened and closed,) could be lengthened or shortened at pleasure, by thrusting in or withdrawing a wire *F f*, which bears with a slight spring against the tongue at *f*.

The blast of wind being maintained constant, the reed was made to yield its gravest note, by withdrawing the wire as far as possible; after which, by pushing it in, the pitch of the reed was gradually raised. It was observed, then, that the tone of the pipe grew constantly more acute; but, after a certain point, it began to diminish in intensity, till at length no sound could be heard. At this point, the tongue of the reed, being narrowly examined through the glass, was observed to be still in rapid vibration; but its vibrations were performed entirely in the air, so as not to strike upon and close the orifice. A constant passage, then, being left for the air, the vibrations of the pipe could not be excited.

But this state of things continued only so long as the tongue was of that precise length. The moment the wire was pushed in by the smallest quantity, the sound sprung forth anew of a pitch still corresponding with the shortened state of the tongue.

89. The influence of the air in a pipe on the reed, by which it is set in vibration, causes the *quality* of the tone of a reed-pipe to depend materially on its figure.

Thus it is found that a reed-pipe of the funnel-shaped form, (fig. 24,) composed of two cones, one more divergent than the other, set on an orifice, gives the clearest and most brilliant tone; but, on the other hand, if the upper cone be reversed, so as to contract the aperture, (fig. 25,) the sound is stifled. But when two similar cones, placed base to base, are adapted to the aperture of a long conical pipe, as in fig. 26, the sound acquires remarkable fullness and force. This belongs, however, to a most intricate part of the theory of sound, the vibrations of masses of air in cavities of any form.

90. The quality of the tone produced by reed-pipes will also of course materially depend on the construction of the reed itself, and the substance of which it is composed.

If the vibrating lamina be of metal, and at every vibration it strikes on a metallic orifice, these blows will be heard, and will give a harsh, rude, and screaming character to the sound. If the edges of the aperture be covered with soft leather, this is much alleviated.

But if, instead of *covering* the aperture by *striking on* it, the tongue is so constructed as merely to *obstruct* it, by passing backwards and forwards *through* it at each oscillation, care being taken to make it *fit* without touching the edges of the aperture, these blows are avoided altogether; the tongue coming in contact with nothing but air during its whole motion. In consequence, its tone is remarkably soft and pure, and free from any harshness.

The invention of this reed is ascribed by Biot to M. Grenié, who has taken out a patent for it; but, without erecting a prior claim on the part of Kratzenstein, we may bring forward a very familiar instrument, the Jew's-harp, as offering, at least, an apparent analogy with M. Grenié's reed.

91. The construction of the Jew's-harp is so well known that there is no need to describe it; and

although the theory of of it be somewhat obscure, there can be little doubt that its action is that of a reed, which calls into play the resonance of the cavity of the mouth, and sympathizes with it in its vibrations, at least in some of their modes.

This instrument is much mistaken and unjustly contemned. Nothing can exceed the softness, sweetness, and delicacy of it, when carefully constructed and played, as by M. Eulenstein; and this might be expected from a reed in which the tongue is perfectly at liberty.

That the instrument itself vibrates in unison with the note it calls forth, is evident from the fact, that when merely held before the open mouth, or lightly retained between the lips, its sound is feeble and scarce audible, but acquires a great accession of force, when brought in contact with and firmly held between the teeth; the note is still further sustained and reinforced by directing a current of air forcibly through it. It is not here meant to say, that the great oscillations to and fro of the tongue are commanded by the resonance of the cavity, or are performed in the same time with its vibrations. On the contrary the spring is far too strong and large to admit of this. It is more probably by a series of subordinate vibrations, going on in the tongue while oscillating, that the sympathy is established.

92. The instrument called the German harmonica is a reed, on M. Grenié's principle, consisting of nothing but a very thin lamina of brass, of the form of an oblong parallelogram, fixed by one of its narrow ends in a frame of its own shape, but just so much larger as to allow of its free motion.

This instrument vibrates by a blast urged through it, yielding a clear musical tone of a very pleasing character and fixed pitch.

If placed at the end of a pipe, it performs the office of a reed, and its tone commands, or is commanded by the pipe according to circumstances, as above explained.

Case where the Pipe is commanded by the Embouchure.

93. When the action of the embouchure of a pipe is so decided as to be incapable of being, to any sensible extent, commanded or influenced by the resonance of the pipe; as, for instance, when the column of air in a stopped pipe is set in vibration by a tuning-fork furnished with a disc, as in fig. 17, the pipe will sound, and reinforce the sound of the tuning-fork, but more and more feebly, as the pitch of the latter departs more and more from that of the pipe.

The experiment is easily made by tuning the upper joint of a flute, with the mouth-hole stopped, exactly in unison with a fork, and then moving the piston of cork at the end of the pipe to and fro, or loading the fork with wax, so as to put it more or less out of tune. The fork and aerial column vibrate as a system, in which the former has so much the preponderance as to command the latter completely.

94. The preceding principle, combined with the principle of the *superposition* of vibrating motions and the simultaneous coincidence of different modes of vibration in the same vibrating body, is curiously illustrated in a remarkable experiment, by which a pipe is made to yield, at the same time, two different and discordant notes.

If, instead of one, two disked tuning-forks be held over the mouth of a pipe side by side, both nearly in unison with the pipe, but purposely tuned out of unison with each other, by an interval so small as to produce strong *beats*, (see the following chapter on beats;) *both* sounds at once will be reinforced by the pipe; and the beats will be heard with the same degree of distinctness, as if two pipes, each in unison with one of the forks, were sounding side by side. The same column of air, then, at the same time, is vibrating as a part of two distinct systems; and each series

of vibrations, however near coincidence they may be brought, continues perfectly distinct and absolutely free from any mutual influence. To those, who have not tried the experiment, the fact of a pipe actually out of tune with itself, and yielding two notes in irreconcilable discord with one another, yet both equally clear and loud, will, at first sight, appear not a little extraordinary.

95. One of the most singular species of pipe is that employed in the organ to imitate the human voice.

It is composed of a very short conical pipe, (fig. 27,) the base upwards, surmounted by a short cylinder, and the pitch is regulated entirely by the reed. There is a circular operculum which half closes the open end of the cylinder, to imitate the lips, the reed performing the part of the larynx, and the pipe itself, that of the cavity of the throat and mouth.

This pipe, when well executed, imitates the human voice extremely well; but with a peculiar nasal twang, and somewhat of a screaming tone.

96. Chimney-pipes, (fig. 28,) are those which are closed at the upper end by a cover, through the centre of which a pipe of smaller diameter is passed, as a continuation of the lower one. Their sound is intermediate between those of open and stopped pipes of the same length.

CHAPTER IV.

MUSICAL INTERVALS, HARMONY, AND TEMPERAMENT.

97. Our appreciation of the *pitch* of a musical sound depends, as we have seen, entirely on the number of its vibrations performed in a given time. Two sounds, whose vibrations are performed with equal rapidity, whatever be their difference in quality or intensity, affect the ear with a sentiment of accordance which we term a *unison;* and which irresistibly impresses on us the conviction of a perfect analogy, or similarity between them, which we express by saying that their *pitch* is the same, or that they sound the same *note.*

In fact their impulses on the air, and on the ear, through its medium, occurring with equal frequency, blend, and form a compound impulse, differing in quality and intensity from either of its constituents, but not in the frequency of its recurrence; and, therefore, the ear will judge of it as of a single note of intermediate quality.

98. But when two notes not in unison are sounded at once, the ear distinctly perceives both, and, (at least with practice, and some ears more readily than others,) can separate them, in idea, and attend to one without the other.

But besides this, it receives an impression from them jointly, which it does not acquire when sounded singly, even in close succession, an impression of con-

cord, or dissonance as the case may be; and is irresistibly led to regard some combinations as peculiarly agreeable and satisfactory, and others as harsh and grating.

99. Now it is invariably found that the agreeable sounds are those, and those only, in which the vibrations of the individual notes are in some very simple numerical proportion to each other, as 1 to 2, 1 to 3, 1 to 4, 2 to 3, &c.; and that the concord is more satisfactory and more pleasing, the lower the terms of the proportion are, and the less they differ from each other.

While, on the other hand, such notes as vibrate in times bearing no simple numerical ratio to each other, or in which the times of the ratio are considerable, as 8 : 15, for example, when heard together produce a sense of discord, and are extremely unpleasant. This simple remark is the natural foundation of all harmony.

a. Next to a *unison*, in which the vibrations of the two notes are as 1 : 1, the most satisfactory concord is the *octave*, where the vibrations are as 1 : 2, or one note performs two vibrations to each single one of the other. The *octave* approaches in its character to a unison; and, indeed, two notes so related, when played together, can hardly be separated in idea; and, when singly, appear rather as the same note differently modified, than as independent sounds. The reason of this will be evident on inspecting fig. 29, where the dots in the upper line represent the periodically recurring impulses on the ear produced by the vibrations of the acuter note, while those in the lower represent the same impulses as produced by those of the graver. As the ear receives these all in the order they are placed, it will be the same thing as if they were produced

by two sounds, both of the graver pitch, but one of a different intensity and quality from the other; the one having its impulses (represented by :) the sum of two separate impulses of the octave sounds; the other consisting of the alternate impulses (.) of the acuter only.

b. In like manner the *octave* of the *octave*, or the *fifteenth*, as it is called in music, which consists of notes whose vibrations are as 1 : 4, is a very agreeable and perfect concord; as are, indeed, all the scale of octaves 1 : 8, 1 : 16, &c. They all partake of the peculiar character of the octave, a sense of perfect adjustment or identity.

c. The next in order is the combination 1 : 3, where the vibrations of the graver note are trisected by those of the acuter, as in fig. 30, which gives a concord called the *twelfth*, a very agreeable one.

In this, if we substitute for the note 1 its octave 2, we shall have the concord whose vibrations are in the ratio of 2 : 3; or, as we shall call it for brevity, the concord 2 : 3, whose pulsations are represented in fig. 31. This concord is termed the *fifth*, and is a remarkably perfect and agreeable one, even more so than the twelfth, which, although simpler in a numerical estimate, yet from the greater interval between its component notes, allows them to be more readily distinguished, while the notes of the fifth blend much more perfectly.

If, instead of substituting for 1 its octave 2, we substitute its double octave 4, we get the concord 4 : 3, (fig. 32,) or the *fourth*, which may be regarded as a sort of complement of the fifth, and is also very agreeable.

d. The concords 1 : 5, 2 : 5, and 4 : 5, (fig. 33,) especially the latter, in which the tones approach pretty near to each other, are all remarkably agreeable. The last is called a *major third*, and the two former are regarded rather as varieties of it than as independent concords, and are called the *tenth* and the *seventeenth.*

The concord 8 : 5, (fig. 33,) (which is the complement to 4 : 5 in the same sense as 4 : 3 the fourth is to 2 : 3 the fifth,) is called the *minor sixth*, and, is almost equally agreeable with the major third, to which it is related.

The concord 3 : 5, (fig. 34,) is called the *major sixth*, and, as well as its complement 6 : 5, or the *minor third*, though pleasing, is decidedly less satisfactory than the foregoing; and, as we see by casting our eyes on the figure, the periods of recurrence of their combined pulses in the same order is longer and more complex.

e. Higher primes than 5 enter into no harmonic ratios. Such combinations, for instance, as 1 : 7, 5 : 7, or 6 : 7, are altogether discordant. The same may be said of the more complicated combinations of the lower primes 1, 2, 3, 5. The ear will not endure them and cannot rest upon them.

100. When a discord is sounded, a sense of craving for a change is produced, and this is not satisfied but by changing one or both of the notes so as to fall, as easily as the case will permit, into some one of the concords above enumerated. This is called the resolution of a discord; and such is the constitution of our minds in this respect, that a concord agreeable in itself is rendered doubly so by being thus approached through a discord.

For example, let us take the ratio 5 : 9, which is called a *flat seventh*, a combination decidedly discordant. If we multiply the terms of this ratio by 5, we get 25 : 45. A small change in one of the notes will reduce this to 27 : 45, or 3 : 5, a major sixth, an agreeable concord.

Now this will be done, if, retaining the lower note 5 or 25, we change the upper one from 45 to 45 $\times$ $\frac{25}{27}$; that is to say, to a note whose vibrations are to its own as 25 : 27. This ratio corresponds to a musical interval called a *semitone*. Hence the discord

in question will be satisfactorily resolved by holding on its lower note, and making its upper one *descend a semitone.*

101. On the proper alternation of concords and discords the whole of musical composition depends; but though the principle above stated must be satisfied in the resolution of every discord, there are other rules to be attended to, by which our choice is limited to some modes rather than others.

For example, in the foregoing instance it is the upper note which must descend a semitone. The ascent of the lower by the same interval, which would equally change the ratio as above indicated, would offend against other precepts with which we have here nothing to do.

102. The *interval*, as it is called in music, between the two notes of which any simple concord or discord consists, depends not on the absolute number of vibrations which either makes in a given time, but on their relative proportion.

For it is no matter how slowly, or how rapidly, the vibrations take place, provided the order in which their impulses reach the ear be the same. Hence, if the vibrations 4 and 5 produce on the ear the agreeable effect of a major third; two notes, each an octave higher, or having their vibrations respectively 8 and 10; or, in general, any two, having their vibrations in this ratio, will produce the same effect.

This is a matter of experience; but the inspection of the figures representing the order of succession of the individual vibrations enables us to understand its reason.

103. The *diatonic*, or *natural* scale is a series of notes, which all ages and nations have agreed in

adopting as the foundation of their music; and, with the vibrations of its respective notes, it stands as follows:

Signs . . .	(1).	(2).	(3).	(4).	(5).	(6).	(7).	(8).
Names of Intervals . .	1st.	2d.	3d.	4th.	5th.	6th.	7th.	8th.
Ratios of Vibration . .	1,	$\frac{9}{8}$,	$\frac{5}{4}$,	$\frac{4}{3}$,	$\frac{3}{2}$,	$\frac{5}{3}$,	$\frac{15}{8}$,	2.

or multiplying all by 24, to avoid fractions,

24, 27, 30, 32, 36, 40, 45, 48.

This scale consists of seven distinct notes; for the eighth being the octave of the first is regarded as a mere repetition of it. And if we add to it on both sides the octaves of all its tones above and below, and again the octaves of these, and so on, we may continue it indefinitely upwards and downwards.

When all its notes are sounded in succession, whether upwards or downwards, the effect is universally acknowledged to be pleasing. The ear rests with perfect satisfaction on the fundamental note, and the intervals succeed each other gracefully, with sufficient variety to avoid monotony.

a. If we take any note for a fundamental sound, and tune a string or a pipe so as to vibrate with the degree of rapidity corresponding to that sound, and represent by unity the number of vibrations it makes per second; and if we also tune other strings to make in the same time respectively the numbers of vibrations represented by $\frac{5}{4}$, $\frac{4}{3}$, $\frac{3}{2}$, $\frac{5}{3}$, 2; and then sound all these strings in succession, beginning with the fundamental note; we shall perceive that two of the sequences, the first and last, are much wider than the rest, and would admit of the interpolation of a note between each. This series is the same as that of the diatonic scale deprived of its second and seventh notes.

b. It is no longer possible to choose for these interpolated notes such as will make concordant intervals with any of the rest, or their octaves. But in order to obtain as many concords as possible in the scale, so as to produce the most harmonious music, they are made to harmonize with that note which bears the nearest relation to the fundamental one, (for its octave is regarded as a mere repetition of itself,) i. e. the fifth.

c. The vibrations of a note a fifth higher than the fifth are represented by $\frac{3}{2} \times \frac{3}{2}$, or $\frac{9}{4}$; and as this is greater than 2, it lies beyond the octave. We must therefore tune our interpolated string an octave lower, or to the vibration $\frac{9}{8}$, and thus get the *second.*

d. The vibrations of a note greater than the fifth by a major third, which is the next most harmonious interval on the scale, are represented by $\frac{3}{2} \times \frac{5}{4}$ or $\frac{15}{8}$; and this represents the *seventh.*

104. Although the diatonic scale may be continued indefinitely, the ear will not follow its additional tones to an unlimited extent. But the whole range of human hearing, comprised between the lowest notes of the organ and the highest known cry of insects, seems to include about nine octaves.

a. When the vibrations are less numerous than about 16 per second, the ear loses the impression of a continued sound; and perceives, first, a fluttering noise, then a quick rattle, then a succession of distinct sounds capable of being counted.

b. On the other hand, when the frequency of the vibrations exceeds a certain limit, all sense of *pitch* is lost; a shrill squeak, or chirp, only is heard; and what is very remarkable, many individuals, otherwise no way inclined to deafness, are altogether insensible to very acute sounds, even such as painfully affect others. This singular observation is due to Dr. Wollaston.

Nothing can be more surprising than to see two persons, neither of them deaf, the one complaining of the penetrating shrillness of

a sound, while the other maintains there is no sound at all. Thus while one person mentioned by Dr. Wollaston could but just hear a note four octaves above the middle E of the piano-forte, others have a distinct perception of sounds full two octaves higher. The chirp of the sparrow is about the former limit; the cry of the bat about one octave above it; and that of some insects probably more than another octave.

c. It is probable, however, that it is not alone the *frequency* of the vibrations which renders shrill sounds inaudible. There is no reason why an impulse, if strong enough *singly* to affect the ear, should lose its effect if repeated many thousand times in a second. On the contrary, such repetition would render the noise intolerable.

But this is not the case with musical sounds; their individual impulses would, probably, be quite inaudible singly, and only impress by repetition. Now, as vibrating bodies have only a certain degree of elasticity, extreme frequency of vibrations can only take place when their dimensions are very minute; and consequently the excursions of their molecules from rest, and their absolute velocities, excessively minute also.

Thus, in proportion as sounds are more acute, their *intensity*, which depends wholly on the extent and force of their vibrations, diminishes. No doubt, if by any mechanism a hundred thousand hard blows per second could be regularly struck by a hammer on an anvil, at precisely equal intervals, they would be *heard* as a most deafening shriek; but in natural sounds the impulses lose in intensity more than they gain in number, and thus the sound grows feebler and feebler till it ceases to be heard.

105. The limits of the sense of hearing possibly, indeed probably, vary with different animals.

As there is nothing in the nature of the atmosphere to prevent the existence of vibrations incomparably more frequent than any of which we are conscious, we may imagine that animals like the Grylli, whose powers appear to commence nearly where ours terminate, may have the faculty of hearing still sharper sounds, which

we do not know to exist; and that there may be other insects hearing nothing in common with us, but endued with a power of exciting, and a sense which perceives vibrations, of the same nature indeed as those which constitute our ordinary sounds, but so remote that the animals who perceive them may be said to possess another sense, agreeing with our own solely in the medium by which it is excited.

The same may, no doubt, be true of aquatic animals. The shrimp and the whale may have no sound in common. Spiders are said to hear the sound of music.

106. By the aid of the ascending and descending series of sounds in the natural scale thus obtained, pieces of music perfectly pleasing, both in point of harmony and melody, may be played; and they are said to be in the *key* of that which is assumed as the fundamental note of the scale, or whose vibrations are represented by 1.

If such a piece be analyzed, it will be found to consist entirely, or chiefly, of triple and quadruple combinations, or chords, such as the following:

First. The common or fundamental chord, or chord of the *tonic*, or the 1st, 3d, and 5th, (1, 3, 5,) or the 3d, 5th, and octave, (3, 5, 8,) sounded together.

Secondly. The chord of the *dominant*, or the notes (2, 5, 7,) sounded together.

Thirdly. The chord of the *sub-dominant*, or the combination (1, 4, 6.)

Fourthly. The *false close*, or the combination (1, 3, 6,) or (3, 6, 8.)

Fifthly. The discord of the 7th, or (2, 4, 5, 7.)

a. The common chord is the most harmonious and satisfactory chord in music, and when sounded the ear is satisfied, and requires

nothing further. It is, therefore, more frequently heard than any other, and its continual recurrence in a piece of music determines the *key* it is played in.

b. The fifth of the key-note is called, by reason of its near relation to the fundamental note, the *dominant.* The chord of the dominant is, then, the *common* chord of the dominant.

c. Note (4) is the *sub-dominant,* and its *common* chord is the chord of the sub-dominant.

d. The false close is the common chord of the note (6), only with a minor third instead of a major. The term *false close* arises from this, that a piece of music, frequently before its final termination, (which is always on the fundamental chord,) comes to a momentary close on this chord, which pleases only for a short time, but requires the strain to be taken up again and closed as usual to give full satisfaction.

e. The discord of the 7th consists of four notes; and is in fact the common chord of the dominant with the note immediately below it, or the seventh *in order* above it. The interval, however, between the notes (4) and (5), or between (5) and the octave of (4) next above it, is represented by the ratio,

$$\tfrac{8}{3} \div \tfrac{3}{2} = \tfrac{16}{9},$$

or (taking 24 as the number of vibrations in a unit of time corresponding to note (1)) $= 42\tfrac{2}{3}$. This interval, then, is less than the seventh of the diatonic scale, and is about half-way intermediate between the sixth and seventh of that scale. It is, therefore, called the flat seventh. (See article 108.)

This discord resolves itself into the chord (3, 5, 8); and unless that combination, or one equivalent to it, follows, the ear is not satisfied. The notes (4) and (5) are the essential ones of this discord, and the others are regarded as accompaniments. If played together, the ear requires that in the next chord (4) should descend or be succeeded by (3), while the note (7) is required to rise or be succeeded by (8.)

107. With these chords and a few others, such as the chord of the 9th, whose essential notes are (1) and (2), or (1) and (9), may a great variety of music be played, but it would be found monotonous. The ear requires, in a long piece, a variety of *key*. The fundamental note occurs so often, that it seems to pervade the whole of the composition, and must therefore be changed. But this change of key, which is called *modulation*, is not possible without introducing other notes than those already enumerated.

In order to play equally well in all keys, every note must have others which differ from it by the intervals of a second, a third, a fourth, &c. But this is not the case in the diatonic scale. Thus the number of vibrations in a unit of time, corresponding to the major third of the note (2), is $27 \times \frac{5}{4} = 33\frac{3}{4}$, nearly half way between those of note (4) and note (5). A new note would therefore have to be introduced, and similarly for other ratios or other notes.

But this would require an enormous number of notes, and would render the generality of musical instruments too complicated. It becomes necessary, then, to consider how the number may be reduced, and what are the fewest notes that will answer.

108. The principle, on which the reduction of the number of notes is made, is, that, if two notes differ from each other by only one vibration in 80, the ear can hardly perceive the difference between them, and the substitution of one of them for the other will not be fatal to harmony.

a. The interval between two such notes is $\frac{81}{80}$, and is called a *comma*.

b. As an example of this principle, we will apply it to the re-

search of the fifth of note (2). The number of vibrations corresponding to the fifth of this note is

$$27 \times \tfrac{3}{2} = 40\tfrac{1}{2},$$

which only differs by a comma from 40, the vibrations of note (6); and, therefore, note (6) may be used as the fifth of note (2), or as the dominant when (2) is the key-note.

109. Some new notes must, however, be interpolated; and, when any note is introduced between two others of the scale, it is denoted in music by the sign $\sharp$ *sharp*, or $\flat$ *flat*.

Thus the major third of the note (2) is, as in art. 107, nearly half way between note (4) and note (5), and must, therefore, be interpolated. It is written either as (4) *sharp*, (4)$\sharp$, or as (5) *flat*, (5)$\flat$.

110. The deficiencies of the diatonic scale are nearly supplied by the interpolation of a new note half-way between each of the larger intervals of the scale, thus

$$1,\ \left.\begin{matrix}1\sharp\\ \text{or}\\ 2\flat\end{matrix}\right\},\ 2,\ \left.\begin{matrix}2\sharp\\ \text{or}\\ 3\flat\end{matrix}\right\},\ 3,\ 4,\ \left.\begin{matrix}4\sharp\\ \text{or}\\ 5\flat\end{matrix}\right\},\ 5,\ \left.\begin{matrix}5\sharp\\ \text{or}\\ 6\flat\end{matrix}\right\},\ 6,\ \left.\begin{matrix}6\sharp\\ \text{or}\\ 7\flat\end{matrix}\right\},\ 7,\ 8;$$

and the scale so interpolated is called the chromatic scale.

111. Musicians have long been at issue on the most advantageous method of executing this interpolation; and it is found necessary to depart from the pure and perfect diatonic scale, even in tuning the *natural* notes. To do so with the least offence to the ear is the object of a perfect system of *temperament*.

a. If, indeed, it were intended to give such a preference to the natural scale 1, 2, 3, 4, &c. as to make *it* perfect, to the sacrifice

of all the other keys, there would be little difficulty; as a mere bisection of the intervals would, probably, answer every practical purpose. Thus 1♯ or 2♮ might be represented by $\sqrt{(1 \times \frac{9}{8})}$; 2♯ = 3♮ by $\sqrt{(\frac{9}{8} \times \frac{5}{4})}$ and so on.

b. But, in practice, no preference is given to this particular key, which is denoted in music by the letter *C*; on the contrary, variety is purposely studied. If the ear absolutely required perfect concords, there could be no music, or but a very limited and monotonous one. But this is not the case. Perfect harmony is never heard, and if heard would probably be little valued, except by the most refined ears; and it is this fortunate circumstance which renders musical composition, in the exquisite and complicated state in which it at present exists, possible.

112. The limits, to which the ear will bear a deviation from exact consonance of musical vibrations, may, however, be judged of by the alternate reinforcements and subsidences of the sound called *beats*, which take place when two notes nearly, but not quite, in unison or concord are sounded together.

The nearer the sounds of the strings approach to exact unison, the longer is the interval between the beats. When the unison is complete, no beats are heard.

On the other hand, when it is very defective, they have the effect of a rattle of a very unpleasant kind.

The complete destruction of the beats affords the best means of attaining by trial a perfect harmony.

a. Conceive two strings, exactly equal and similar, and equally drawn out from the straight line, to be let go at the same instant; and suppose one to make 100 vibrations per second, the other 101; let them be placed side by side, and at the same distance from the ear.

Their first vibrations will conspire in producing a sound-wave of double force, and the impression on the ear will be double.

But, at the 50th vibration, one has gained half a vibration on the other, so that the motions of the aerial particles, in virtue of the two coexistent waves emanating from either string, are not now in the same, but in opposite, directions; and the two waves, being by supposition of equal intensity, will, instead of conspiring, exactly destroy each other; and this will be very nearly the case for several vibrations on either side of the 50th. Consequently, in approaching the 50th vibration, the joint sound will be enfeebled; there will be a moment of perfect silence, and then the sound will again increase till the 100th; when the one string having gained a whole vibration on the other, the motions of the particles of air in the two waves will again completely conspire, and the sound will attain its maximum.

b. If we call n the number of vibrations, in which one string gains or loses exactly one vibration on the other, and m the number of vibrations per second made by the quicker, $\frac{n}{m}$ will be the interval between two consecutive beats.

c. Beats will likewise be heard when other concords, as fifths, are imperfectly adjusted.

Suppose one string to make 201 vibrations, while the other makes 300; then, at and about the 100th of one, and the 150th of the other, the former will have gained half a vibration, and those vibrations of the one, which fall exactly on those of the other, (see fig. 31,) being performed with contrary motions, will destroy each other; those which fall intermediate only partially. The beats will then be heard, but with less distinctness than in the case of unisons.

113. We may here notice an effect which takes place in perfect concords, and only in those which are very perfect, viz., the production of a grave sound by the mere concurrence of two or more acute ones.

a. If we examine the figure 31, which represents the succession of vibrations in a perfect fifth, we shall see that every third of the one coincides exactly with every second vibration of the other. These coincidences, (so delicate is the ear,) are remarked by it, and a sound is heard, besides the two actually sounded, of a pitch determined only by the frequency of the precise coincidences; that is, in this case, a precise octave below the lowest tone of the concord.

b. In general, if one note makes m vibrations and the other n, while another, of which they may be both regarded as harmonics, makes one, that one will be the resultant tone, provided m and n be prime to each other; so that the only difficulty, in determining the resultant of two notes, is to determine of what they are both harmonics.

This will be done by reducing m and n, if fractions, to a common denominator $\frac{m'}{N}$ and $\frac{n'}{N}$; then, if m' and n' have no common factor, $\frac{1}{N}$ will represent the fundamental tone. If, then, m and n be integers, and without any common factor, the resultant will be represented by 1.

c. Hence follows a very curious fact, viz., that if several strings, or pipes, be tuned *exactly* to be harmonics of one of them, or to have their vibrations in the ratios 1, 2, 3, 4, 5, &c., then, if they be all, or any number of them, from the first onward, sounded together, there will be heard but one note, viz., the fundamental note.

For they are harmonics of the first note 1; and, moreover, if we combine them all two and two, we shall find comparatively but few which will give other resultants, so that these will be lost, as well as the individual sounds of the strings, all but the first, in the united effect of all the resultant unit sounds.

But to produce this effect, the strings, or pipes, must be very perfectly tuned to be strict harmonics. The effect can never take place by touching the keys of a piano-forte corresponding to the harmonic notes, because they are always of necessity tempered.

114. To return to the subject of temperament; the most simple method of distributing the imperfections of the chromatic scale is, to make the successive intervals between the notes all equal; and the scale thus obtained is called the *scale of equal intervals.*

This scale must, obviously, be the result of a *system of equal temperament*, which consists in making all the octaves perfect, and all the fifths and thirds equally imperfect.

a. If we count the semitones in the chromatic scale between (1) and (8), we shall find the number of such intervals 12. If, then, we would have a scale exactly similar to itself in all its parts, and which would admit of our playing equally well in every key; we have only to divide the whole interval from (1) to (8) into 12 equal parts. But, as the interval between two notes is nothing but the ratio, or quotient of the number of vibrations of the second divided by those of the first; these ratios are here all equal, or the numbers of vibrations form a geometrical series, whose first term is 1, last term 2, and number of terms $12 + 1 = 13$. The following must, therefore, be the series,

$$1 = 2^0,\ 2^{\frac{1}{12}},\ 2^{\frac{2}{12}},\ 2^{\frac{3}{12}} \ldots\ldots\ 2^{\frac{11}{12}},\ 2;$$

and the values of the fractions may be computed by logarithms, and we shall thus obtain the scale of equal intervals.

b. The impossibility of forming a scale, exactly similar to itself in all its parts, of 12 or any other limited number of notes, in which the fifths and thirds, as well as the octaves, should all be perfect, appears from the consideration; that, if we start from any note and obtain its fifth and then the fifth of this fifth, and again the fifth of this new note, and so on indefinitely, we shall never arrive at a note contained among the octaves of either of the notes before obtained; but shall form an endless series of *new* notes which must all be inserted in the scale. The series of fifths will, indeed, be

$$\tfrac{3}{2},\ \tfrac{3}{2} \times \tfrac{3}{2} = (\tfrac{3}{2})^2,\ (\tfrac{3}{2})^2 \times \tfrac{3}{2} = (\tfrac{3}{2})^3,\ (\tfrac{3}{2})^4 \ldots \text{\&c.},$$

and will consist of the different powers of $\frac{3}{2}$, while the series of octaves consists of the different powers of 2; but, since no power of 2 is exactly the same with any power of $\frac{3}{2}$, it is evident that no note can be found in one series which is contained in the other.

c. Theoretically speaking, the iso-harmonic scale is the simplest that could be devised; and, practically, though fastidious ears may profess to be offended by it, it must produce no contemptible harmony. It has, however, one radical fault; it gives all the keys one *character*. In any other system of temperament some intervals, though of the same denomination, must differ by a minute quantity from each other; and this difference, falling in one part of the scale in one key, in another in another key, gives a peculiarity of quality to each key, which the ear seizes and enjoys extremely.

This fact, in which, we believe, all practical musicians will agree, is alone sufficient to prove, that *perfect* harmony is not necessary for the full enjoyment of music. Most practical musicians seem to have no fixed or certain system of temperament; at least very few of them, when questioned, appear to have any distinct ideas on the subject.

115. It is a mistake to suppose, as some have done, that temperament applies only to instruments with keys and fixed notes. Singers, violin-players, and all others, who can pass through every gradation of tone, must all temper, or they could never keep in tune with each other or with themselves.

Any one, who should keep on ascending by perfect fifths, and descending by octaves or thirds, would soon find his fundamental pitch grow sharper and sharper, till he could at last neither sing nor play; and two violin players accompanying each other, and arriving at the same note by different intervals, would find a continual want of agreement.

116. Many different systems of temperament have been proposed; and some fruitless attempts have been

made to reduce the subject to arithmetical principles; but it remains a mere matter of individual taste. It has generally been considered preferable, however, to preserve some keys more free from error than others; partly for variety, and partly because keys with five or six sharps or flats are comparatively little used, so that these may safely be left more imperfect, (which is called by some, throwing the *wolf* into these keys.)

a. A variety of systems of temperament have been devised for producing the best harmony by a system of 21 fixed sounds, viz. each note of the seven in the diatonic scale with its sharp and flat. The first and most celebrated is that of Huygens; and an exceedingly refined system has also been proposed by Dr. Smith. Either system, no doubt, will give very good harmony; but as on the piano-forte only 12 keys can be admitted, and as this instrument is now become an essential element in all concerts, and indeed the chief of all, a temperament *must* be devised which will accomodate itself to that condition.

b. The system called by Dr. Smith that of mean tones, or the vulgar temperament, though the most inartificial, is probably as good as any which the nature of music admits, holding a sort of mean between the advantages and defects of all the rest. In this system, the third (3) of the diatonic scale is perfect, and the fifth (5) is tempered a little flat, flatter than a perfect fifth by a quarter of a comma. Note (2) is half-way between (1) and (3); (6) is the fifth of (2) and the third of (4); and (7) is the third of (5). The sharps and flats of the chromatic scale are inserted by bisecting the larger intervals.

c. Mr. Logier endeavoured to place the interpolation of the intermediate notes between those of the natural scale on *à priori* grounds, by assuming the flat seventh (7)♭ as the seventh *harmonic* of the fundamental note (1); that is to say, the note produced by subdividing into seven equal parts the length of a string whose

fundamental tone is (1), or at least one of the octaves of that note. But sevenths, tuned on his principle, will require a much more violent temperament than either fifths or thirds, either of which *might* be used as a means of introducing the intermediate notes; and the system must in consequence be abandoned, as must every system which professes to render musical arithmetic any thing more than a matter of convention and approximation.

CHAPTER V.

OF THE SONOROUS VIBRATIONS OF BARS, RODS, AND PLATES.

117. THE vibrations of all bodies, if of a proper degree of frequency, and of sufficient force to be communicated through the air, or any other intermedium, to our organs of hearing, produce sounds whose pitch depends on their frequency; and their force and quality on the extent, or other mechanical circumstances of the vibrations, and the nature of the vibrating body. The mathematical investigation of these vibratory motions is altogether foreign to our purpose; it is one of the most intricate and least manageable branches of Dynamics.

118. A solid body may vibrate, either in consequence of its inherent elasticity, by which it tends to return to its own proper figure and state, when forcibly deranged, or in consequence of an external tension.

To the former sort of vibrations belong those of rods, tuning-forks, plates, rings, bells, gongs, and vessels of all shapes, or generally, of all solid masses which *ring* when struck. To the latter, those of vibrating strings and membranes, such as the parchment of a drum or a tambourine, &c.

119. But, further, a solid may vibrate by its own proper elasticity in two very different ways.

First, an undulation may be propagated through it, as through an elastic compressible medium; and in this case, the waves will consist of alternate strata of condensed and rarefied solid matter, precisely similar to those of an elastic fluid, the laws of motion in different directions varying with the varying elasticity of the solid.

If the solid be homogeneous, such as the metals, glass, &c., the elasticity being the same in all directions, the waves will be propagated from the centre of disturbance, according to exactly the same laws as in a mass of air of the same shape. But, if crystallized, this may not be the case, or the vibrations, instead of being in the direction of the propagated wave, may be transverse or oblique to it, or may even not be confined to one plane, but may be performed in circles or ellipses.

120. If a straight rod of glass, or a metal, be struck at the end in the direction of its length, or rubbed lengthways with a moistened finger, it will yield a musical sound, which, unless its length be very great, will be of an extremely acute pitch; much more so than in the case of a column of air of the same length.

The reason of this is the greater velocity with which sound is propagated in solids than in air. Thus the velocity of propagation in cast-iron being 10½ times that in air, a rod of cast-iron so excited

will yield for its fundamental note a sound identical with that of an organ pipe $\frac{1}{10\frac{1}{2}}$ of its length stopped at both ends, or $\frac{1}{21}$ of its length if open at one end. See Chapter III. Part II., all the details of which are applicable to the present case.

To such vibrations Chladni, who first noticed them in long wires, has applied the term *longitudinal.*

121. To produce the harmonics of such a rod or wire, it must be held lightly at the place of one of its intended nodes between the finger and the thumb; and the friction must be applied in the middle of one of the vibrating segments.

If the rod be of metal, the friction, which Chladni found to succeed, was that of a bit of cloth sprinkled with powdered rosin; if of glass, the cloth or the finger may be moistened and touched with some very fine sand or pumice powder.

122. It may be observed here, that, generally speaking, a fiddle-bow well rosined is the readiest and most convenient means of setting solid bodies in vibration.

To educe their gravest or fundamental tones, the bow must be pressed hard and drawn slowly; but for the higher harmonics, a short swift stroke with light pressure is most proper.

In all cases, the point intended to be a node must be lightly touched with the finger; and the vibration must be excited in the middle of a ventral segment.

The vibrations of a cylindrical rod or tube so excited are, in general, more complex than in the above case which was analyzed by Chladni.

123. Secondly. By far the most usual species of vibration, executed by solid bodies, is that in which their external form is forcibly changed, and recovered again by their spring.

124. The simplest case is that of a rod, executing vibrations to and fro in a direction transverse to its length.

a. This case has been investigated mathematically by D. Bernouilli and Euler, as also by Riccati; and their results have been compared with those of experiment by Chladni, and found correct.

b. The cases enumerated by Chladni are six in number.

I. When one end of the rod is firmly fixed in a vice or let into a wall, the other quite free. In this case, the curvature assumed by the rod in its vibrations must of necessity have its axis or position of rest for a tangent, as fig. 35.

II. One end *applied* or pressed perpendicularly against an obstacle, the other free. In this case, the *excursions* of the applied end to and fro are prevented by the friction and adhesion to the obstacle, but the axis is not of necessity a tangent. See fig. 36.

III. Both ends free. Fig. 37.

IV. Both ends applied. Fig. 38.

V. Both ends fixed. Fig. 39.

VI. One end fixed, the other applied. Fig. 40.

c. All these cases have been examined by Chladni at length. We shall, however, select only the fourth case where both ends are applied, because it will afford room for an important remark. In this, then, the several modes of vibration corresponding to 1, 2, 3 vibrating or ventral segments of the rod will be as in figs. 38, 41, 42.

Now these are similar to the curves which would be assumed by a vibrating string under the same circumstances of subdivision. But the notes produced are very different. For, whereas in the case of a string the vibrations of the successive harmonics are

represented by 1, 2, 3, &c.; in that of a rod they are represented by the *squares* of these numbers 1, 4, 9, &c., which correspond to *double* the former intervals. In all other cases the series is still less simple.

125. This alone suffices to show the insufficiency of any attempt to establish, as some have wished to do, the whole theory of harmony and music on the aliquot subdivision of a vibrating string.

Had vibrating rods or steel springs (which yield an exquisite tone) been always used instead of stretched cords, such an idea would never have suggested itself; yet no doubt our notions of harmony would have been what they now are. The same remark applies still more forcibly to the modes of subdivision of vibrating surfaces, which in many cases have their harmonics altogether irreducible to any musical scale.

126. The most simple modes of vibration of a rectangular surface are those, which exhibit quiescent or *nodal* lines parallel to one of its edges.

a. A rectangular plate may be regarded as an assemblage of straight rods of equal length, ranged parallel to each other. Supposing such an assemblage all set in vibration similarly and at once, they will retain their parallel juxtaposition during their vibration, and may, therefore, be supposed to adhere and form a plate. Consequently, among the possible series of vibrations of a rectangular plate will be found all those of a rigid rod, and the harmonics will be the same as those of the rod.

b. When the same mode of vibration of different plates is compared, the number of vibrations is inversely as the square of the length of the plate; but increase of breadth occasions no difference in the sound; and the distance from a free end to a nodal line is rather less than half the distance between two nodal lines.

c. Fig. 43 *a.* shows the situations of the nodal lines, when the ends of the plate are free, and the number of the nodal lines is 2, 3, 4, and 5. Figs. 43 *b.* and *c.* are profiles of the preceding, and represent the curvature of each parallel fibre perpendicular to the nodal lines at the two opposite limits of their vibrations. The quantity of motion at each point is indicated by the corresponding ordinate of the curve, and its direction by its situation above or below the horizontal line.

d. It will be convenient to distinguish these states of motion, in which every corresponding point is moving in direct opposition. The first, *b.* will therefore be called the *positive* states of vibration; and the second, *c.* the *negative* states of vibration. When there is an even number of nodal lines, the positive state of vibration may be considered as that in which the motion at the *central* part is above the plane of equilibrium, and the negative, that in which it is below it.

127. The subdivision of a plane surface in vibration by its nodal lines may be rendered visible to the eye, by holding it in a horizontal position and strewing it over with sand; for the sand will be thrown away from the vibrating parts and accumulate on those at rest.

In the preceding cases of the rectangular plate, the sand will then be arranged in straight lines across the plate parallel to its edges, and their distances apart and from the ends of the plate may be measured at leisure.

128. If we suppose two similar surfaces with the same number of nodal lines to be superposed, and both to vibrate in concurrence, i. e. both either positively or negatively, they will mutually assist each other's effects; but if they vibrate in opposing directions, they will destroy each other's motions, and the entire surface will be at rest.

129. When the rectangular surface is a square, it is obvious that it may vibrate in two different rectangular directions, so as to give the same sound, and present the same arrangement of nodal lines. Now, by the principle of the superposition of vibrations, these two modes of vibration may coexist, and produce a compound, in which the position of the nodal lines will be greatly changed; but the number of vibrations, and consequently the *pitch*, will not materially differ from that of the components.

a. If a plate is excited at a point where the motion of each rectangular mode of vibration is at its maximum, in the same direction, and of equal intensity; there is no reason why one mode of vibration should be produced in preference to the other. On calculating the effect of such coexistence, Mr. Wheatstone found that the resultants of these combined modes of vibration, similar in every thing but in their direction with regard to the sides of the plate, gave rise to new nodal lines which accurately corresponded with figures described by Chladni.

b. The principal results of the superposition of two similar modes of vibration are these.

First. The points, where the nodal lines of each figure intersect each other, remain nodal points in the resulting figure.

Secondly. The nodal lines of one figure are obliterated, when superposed, by the vibrating parts of the other.

Thirdly. New nodal points or lines are formed wherever the vibrations in opposite directions neutralize each other.

Fourthly. At all other points the motion is as the sum of the concurring, or the difference of the opposing, vibrations.

c. A primary figure, having an even number of nodal lines, may be superposed two ways, and may consequently give rise to two distinct resultant figures; one, when the central vibrating parts concur; and the other, when they are in opposition.

But if the number of nodal lines in the primary figure is uneven, there can be only one resultant figure.

d. The nodal lines which thus result may be very easily ascertained.

Take, as an example, the first mode of vibration having two parallel nodal lines. This, being superposed in two rectangular directions, and so that the states of vibration are opposing, (fig. 44,) it is obvious that no lines of compensation can exist in the four rectangular segments *a a a a*, as every point included within them is actuated by concurrent motions. But, in all the other rectangles, they must necessarily be formed, as every point within them is affected by two opposing motions; and if the two modes be of equal intensity, the compensations must occur at every point equally distant from the two rectangular nodal lines, each appertaining to a different mode of vibration. The resultant figure will thus be found to consist of two diagonal lines, perpendicular to each other, and passing through the centre of the plate.

But if the two superpositions vibrate in concurrence, (fig. 45,) the rectangles *b b b b* will be free from compensating points; but these will occur in the other rectangles, and form a figure which also consists of diagonal lines.

e. In the same manner the resultant of any two similar modes of vibration with nodal lines, parallel to the sides, may be proved to consist of lines parallel to the diagonals.

130. It is not a necessary condition for the vibrations of a square plate, that the primary nodal lines shall be parallel to a side ; they may also be parallel to a diagonal, or to any line intermediate between a transverse and a diagonal line ; and these vibrations may be distinguished by the term *primary* vibrations.

131. In these cases the superpositions take place according to the following rule.

The *axes* of the superposed modes of vibration must make equal angles with a line parallel to the edges of the plate passing through its centre; for, otherwise, the modes of vibration would not be similar.

a. By the *axis* of a primary mode of vibration is meant a straight line passing through the centre of the plate, and parallel to the nodal lines.

b. Considerations, of the kind already employed, will show that in all these instances the resultant figures consist of lines parallel to the edges of the plate, and that they are always the same in number as the nodal lines of a component mode of vibration, but differently distributed in the two directions, according as the angle of superposition varies.

c. Some of the various primary modes of vibration, transverse, intermediate, and diagonal, and the angles which the nodal lines of two similar figures make with each other when they are superposed, are represented in the first column of Plates 5 and 6. In the second column of these plates are placed the figures resulting from their opposing superpositions; and in the third column those which arise from their concurring superpositions.

132. We obtain by experiment a limited number only of figures, which can be considered the resultants of primary modes of vibration consisting of any given number of oblique lines; but it would seem, that as the various degrees of obliquity are infinite, so there should be an infinite number of resultant figures passing into each other by insensible gradations. By calculation this should be so; but there is a cause of limitation in the circumstance that no resultant figure is maintainable, unless the greatest excursions of the external vibrating parts occur at the edges of the plate.

In the concurring superpositions of eight oblique lines, this condition can only be fulfilled when the angles they make with each other are either 90°, or 143° 8′. In the opposing superpositions of the same number of lines, the condition is fulfilled when the angles at which the lines are inclined are 118° 4′ and 163° 44′.

133. We are not, however, limited to binary superpositions. But four superpositions may coexist, whenever the axis of the primary figure is oblique to the edges of the plate and differs from the diagonal.

a. When the axis of a primary figure corresponds with a diagonal, or is parallel to the edges of the plate, it is obvious that there can be only one other line of equal length, which can be considered as the axis of a similar and isochronous mode of vibration. In these cases, it is evident, therefore, that there can only be two superpositions.

But in every intermediate direction of an axis, as AB (fig. 46,) there are three other lines $A'B'$, $A''B''$, $A'''B'''$, of equal length, which constitute axes of similar modes of vibration; and four superpositions can therefore take place.

b. It would be a tedious and laborious process to ascertain a resultant figure by combining its four component modes of vibration; but the same purpose will be effectually answered by combining them first in pairs, as in art. 131, and then combining two of these first resultants rectangularly together. The fourth and fifth columns of plates 5 and 6 contain the resultants of four superpositions of the primary figures of the first column.

134. All the modes of vibration, of which a square plate is susceptible, are either primary, or can be obtained from primary vibrations of the same sound by successive superpositions.

a. Chladni, to whom is due the sole merit of having discovered the symmetrical figures, exhibited on plates of regular forms when

caused to sound, investigated the vibrations of square plates with great accuracy, and delineated 90 of their different modes of vibration.

b. Mr. Wheatstone was the first to give a correct analysis of these vibrations. He formed all the different combinations arising from two or four superpositions of similar primary vibrations, the number of whose nodal lines did not exceed 12. On comparing his calculated figures with those obtained experimentally by Chladni, the greater number were found exactly to agree. There are, however, some differences which can easily be explained.

c. In the first place, there is an obvious cause of error in delineating figures from experiment, from this circumstance; that the sand accumulates in the spaces where two convex curves are near and opposite to each other, the motion being there very small, so that it is difficult to ascertain whether the curves join, or not.

d. Secondly, inequalities in the plate will sometimes occasion lines which ought to intersect each other, to appear separated curves.

e. Another cause of difference is this; when the lines of one component figure very nearly coincide with those of the other, but without actually doing so, the resultant figure may be such as would arise from their actual superposition, instead of that which accurate calculation would give.

f. A few of the figures delineated by Chladni are irregular resultants formed by the superpositions of dissimilar modes of vibration. These irregular resultants can, however, be formed from the superposition of dissimilar modes of vibration, which give the same sound, and have a maximum point of vibration in common, at which they can be simultaneously excited.

g. Some of Mr. Wheatstone's figures were not among Chladni's; because the near approach of the inclined lines of their primary component figures to parallelism occasions great difficulty in obtaining these figures by experiment.

h. A few of Chladni's figures exceeded the limits of Mr. Wheatstone's calculations.

i. Lastly, some of Chladni's resultants arose from the superpositions of similar modes of vibration of unequal intensity. These are experimentally obtained by varying in a slight degree the places at which the plate is held or touched, from those necessary to determine the corresponding resultant of vibrations equal in intensity; the place at which the bow is applied remaining the same in both cases.

135. The vibrations of homogeneous plates of other forms can also be analyzed and classed, as well as those of square plates; and all the figures will be found to be resultants of very simple modes of vibration, oscillating isochronously, and superposed upon each other; the resultant varying with the component modes of vibration, the number of superpositions, and the angles at which they are superposed.

The vibrations of circular, triangular, hexagonal, elliptic, and semi-circular plates have been investigated experimentally by Chladni, and figs. 47 – 93 exhibit some out of a great variety of nodal figures, to which they give rise in their various modes of vibration.

136. The series of figures, presented by a plate of any homogeneous material, differ from those obtained on a plate of a substance, such as wood, in which the elasticity is not the same in all directions.

Savart has made a series of numerous and accurate experiments on the changes which take place in the sound, and also in the form and position of the figure of the first mode of vibration on

circular plates of wood of similar dimensions, cut in different directions with respect to the three principal axes of elasticity. He has extended his investigations to circular slices of crystals, cut in various directions with respect to their axes, and has obtained in this way much valuable information respecting the structure of bodies, particularly with regard to the laws of their elasticity in different directions.

PART III.

THE COMMUNICATION OF VIBRATIONS AND THE VIBRATIONS OF SYSTEMS.

CHAPTER I.

THE VIBRATIONS OF SYSTEMS.

137. VIBRATIONS confined to one plane, and whose period of recurrence, as well as the plane in which they are performed, and the amplitude of their excursions may be varied at pleasure, can be communicated to any given point of a solid by means of a stretched string, set in motion by a fiddle-bow.

a. The vibrations of the string are necessarily confined to the plane in which the motion of the bow is performed, because any vibratory motion out of this plane is prevented, or immediately stifled by the pressure of the bow; and the plane of its motion may be varied at pleasure, and the amplitude of excursion may be increased or diminished, by a change of pressure and velocity of stroke.

Accordingly, if the vibrating part of such a string be brought to press on a solid not too massive, or if the end of the string be attached to a point in the solid, M. Savart has found that the regularly repeated impulses of the string are transferred to the solid with perfect fidelity.

b. A familiar example of this communication of impulses is found in the violin. In that instrument, (fig. 94,) the strings

which are stretched from end to end of it are divided into unequal parts by the bridge, *A*, on which they all press strongly, and at the same time rest in small notches, so as not to slip laterally on it. The portion, *B*, of the string which lies towards the handle, *C*, of the instrument, is free, and is set in vibration by the bow in its own plane; but that on the other side of the bridge, *D*, is loaded with a mass of horn or whalebone, *E*, to which, all the other strings are also attached, and which, being only *tied* to the woodwork, cannot propagate the vibrations of any one string sounding separately, by reason of the contradictory and unequal tensions of the other three. Thus the bridge is in fact acted on only by the vibrations of that part, *ABC*, of the string which is crossed by the bow, as if it terminated abruptly at its point of pressure, *A*.

These vibrations constantly tend, therefore, to tilt the bridge laterally backwards and forwards, and to press up and down alternately the two little prominences or feet, *F*, *G*, by which it rests on the belly of the violin. It, therefore, sets the wood of the upper face in a state of regular vibration, and this again is communicated to the back through its sides, a peg set up in the inside of the fiddle and through, called the *soul* of the fiddle, or its *sounding-post*.

In consequence, if sand be strewed over the upper surface, it will assume a regular arrangement in nodal lines when the bow is drawn; and the same subdivision is also observed in the wood of the under surface, if the sounding post be exactly placed in the centre of symmetry of the nodal figures.

c. The experiment can hardly be made, however, with a common fiddle, by reason of the convexity of its surface, on which sand will not rest; but if one be constructed with plane boards, or if, abandoning the fiddle, a string be stretched on a strong frame over a bridge, which is made to rest on the centre of a regularly formed plate or disc of metal or wood, strewed with sand, the surface thus set in vibration by the string will be seen to divide itself by regular nodal figures.

138. If the tension or length of the string, thus placed in vibratory communication with a plate, be changed, so as to vary the note it speaks, the nodal figures on the plate undergo a corresponding variation, and the plate *still* vibrates in unison with the string; or, which is the same thing, the two, together with the interposed bridge, form a vibrating system, in which, though the vibrations of the several parts are necessarily very different in their nature and extent, yet they have all the same periods.

This important fact, deduced by Savart from his experiments, confirms the results of Chladni's experiments on the sounds of such thin plates, and shows that they are not, like those of strings, confined to certain fixed harmonics, but, according to the forms of their nodal lines, and the proportions of the vibrating areas in opposite states of excursion, may assume any assigned period; in other words, given the vibrating plate and the pitch, a nodal figure may be described on it, which shall correspond to that pitch, and the plate (with more or less readiness, however,) is always susceptible of such a vibration as shall yield that note and produce that nodal figure. How far this proposition is general, and with what limitations it is to be understood, we shall soon see.

139. Meanwhile this remark, it will be observed, furnishes a complete explanation of the effect of sounding-boards in musical instruments.

It is not, as some have supposed, that there exist in them fibres in every state of tension, some of which are therefore ready to vibrate in unison with any proposed sound, and, therefore, reinforce it. Such a cause could at best produce but a very feeble effect.

It is the *whole* board which vibrates as part of a system with every note, and (as vibrations may be superposed to any extent) the same sounding-board may at once form a part of any number

of systems, and vibrate in unison with every note of a chord. Still some modes will always be more difficult than others, and no sounding-board will be perfectly indifferent to all sounds.

140. The longitudinal vibrations of a rod of glass, excited by rubbing it with a wet cloth, may also be used to excite vibrations in a given point of a solid perpendicular to its surface, by applying its end to it, or cementing it to the solid by mastic.

In this way Chladni applied it to draw forth the sounds of glass vessels, (which, when hemispherical, of sufficient size, and of even thickness, are remarkably rich and melodious,) in an instrument which he called the Euphone, exhibited by him in Paris and Brussels. The principle of this instrument was at the time concealed; but the enigma was subsequently solved by M. Blanc, who on his part independently made the same remark, and applied it to a similar purpose.

141. If the solid, (a circular glass disc for instance,) to which such a vibrating rod or tube is fastened, be of small comparative dimensions, its vibrations are commanded by those of the rod, and the sound yielded will be that of the rod alone; and *vice versâ*, if the disc be large, and the rod small, the note sounded will be that of the disc, which will entirely command the rod; but in the intermediate cases, the note will be neither that of the disc nor the rod separately, but the two will vibrate together as a system, each yielding somewhat to the other.

a. This is a case exactly analogous to that of a reed-pipe, in which the reed and column of air mutually influence each other's note. See art. 88.

This mutual influence of propagated motion, by which two periodically recurring impulses affect each other's period, and force themselves into synchronism, extends to cases where at first sight it would be hardly suspected.

b. Thus Ellicott observed that two clocks fastened to the same board, or even standing on the same stone pavement, beat constantly together, though when separated their ratios were found to differ very considerably; and Breguet has since made the same remark on watches.

c. Thus also two organ-pipes vibrating side by side, if very nearly in unison, will under certain circumstances force themselves into exact concord, as has been observed by Hudlestone, and lately recalled to notice by some experiments made in Copenhagen.

a. The experiment with the disked tuning-fork and pipe, related in art. 93, may here again be referred to.

142. The longitudinal vibrations of a rod have also been used by M. Savart, to communicate vibrations from one solid to another; as, for instance, from the upper to the under of two circular discs, cemented at their centres to the two ends of the rod, at right angles to their planes, as at fig. 95.

143. If the two discs be of the same dimensions and materials, so as to yield, when separately vibrating, the same note; the vibrations of one of them, (the upper for instance,) excited by a bow, will be exactly imitated by the other; and sand strewed over both will arrange itself in precisely the same forms in both discs, and that, into whatever number of vibrating segments that immediately excited be made to subdivide itself.

But if the discs separately do not agree in their tones, the system may yield a tone intermediate, and each being differently forced from its natural pitch, the nodal figures on them will no longer correspond.

a. The state of vibration, into which the molecules of the connecting rod are thrown in such cases, deserves a nearer examination.

For simplicity, let us suppose the discs equal, the rod cylindrical, and the vibration of the system such, that each disc shall subdivide itself into four quadrantæ segments. In this case it is clear that, as the form assumed at any instant by the upper disc is undulated or wrinkled, as represented in fig. 96, the section of the rod in immediate contact with it, and which obeys all its motions, must assume a similar form, and so of all the rest. Thus, if we conceive the rod split into infinitesimal columns, parallel to its axis, all the columns in two opposite quadrants will be ascending, while those in the other two are descending; and thus the two corresponding opposite quadrants of the lower plate will be drawn upwards, while the alternate ones are forced downwards, giving a similar distortion to its figure, and disposing it to a similar vibration only.

b. It will depend on the length of the rod, and the time taken by an undulation to run over its length, compared with that of a vibration of either disc, whether the *phases* of vibration in the two discs shall be the same at the same instant or not. It may happen that, for instance, the quadrant, *DB*, of the upper disc shall have completed its downward motion, and begun to return, before the pulsation propagated through the rod has arrived at the lower disc; and in that case the corresponding quadrants of the two discs will be always in opposite phases of their periodic motion. But the nodal lines will of necessity correspond in both.

c. When the two discs are unequal, the propagation of the pulses through the rod must of course cease to be uniform, and each section of it down its whole length will have its own peculiar

law of form and motion, which it is beyond our power to investigate. In that case its molecules must have lateral as well vertical motions, and its vibrations must be partly *longitudinal* and partly *twisting*, in a way easier imagined than described.

d. If the discs be dissimilar in form, as well as unequal in dimension, the vibrations of the connecting rod will of course be very complicated.

144. These principles have been applied by M. Savart, and apparently with success, to the improvement of violins, and the construction of these delicate instruments on scientific and experimental grounds.

145. In the vibrations of solids of any figure and dimension, and also in those of masses of air or any other elastic fluid, there will, generally, be *nodal surfaces*, all the points of which are at rest.

Where the surfaces *out-crop* or intersect the external surface of the mass, there will be a *nodal line.*

a. It appears from what we have said, that the motions of the molecules of a rod, which commmunicates the vibrations of one disc to another, or, more generally, which vibrates longitudinally by any exciting cause, are not of necessity analogous to those of the air in a cylindrical pipe; at least not to that simple case of the latter vibrations, which we have heretofore considered in Chapter III., Part II. The several transverse sections of such a rod, in the act of vibration, do not necessarily merely advance and recede longitudinally, but may become curves of double curvature; in short, such a rod may be considered, as an assemblage of vibrating discs, ranged along a common axis, along which they may, it is true, be also carried backwards and forwards with a vibratory motion, while at the same time their flexure is changing from convex to concave, and *vice versâ.*

Now it may happen that a point, or a line, (straight or curved,) in any one of such discs, may be advancing in the direction of the axis in consequence of the bodily motion of the whole disc, while, in virtue of its flexure in the act of changing its figure, it may be receding; and this advance or recess may so balance each other, that the point or line shall be at rest. If this be true at one instant, it will be so at all instants; because the vibrations have all one period, and follow the same law of increase and decrease in their phases.

Thus we have a nodal point, or a nodal line; and as each disc, by reason of the law of continuity, must have a similar one, the assemblage of such lines will mark out within the rod a *nodal surface*, dividing it into separate solids, whose molecules on either side of such surface are in opposite phases of their motion.

b. What is here said of rods, applies of course to solids of any figure and dimension; neither is there the slightest reason why it should not apply to vibrating masses of air, or any other elastic fluid. Any such mass may be conceived as cut up into two or more oppositely vibrating portions, pervading it according to certain laws.

146. If fine sand is strewed over the surfaces of solids, the motions of the particles in the act of marking out the nodal lines will easily distinguish such vibrations, as are executed parallel to the surface, which are called *tangential* vibrations, and in which, of course, the surface is not thrown into waves, from such as are at right angles to it, which are called *transverse* vibrations, when the surface itself leaps up and down. Vibrations, compounded of both these, where the surface both swells and falls, and shifts laterally backwards and forwards, are called *oblique* vibrations.

How to Distinguish Normal from Tangential Vibrations.

a. In *transverse* vibrations, the particles of sand *dance*, and are violently thrown up and down over the whole extent of the vibrating portions, till, at length, they are entirely dispersed from them.

When a large disc of glass is set vibrating vigorously by a bow, perpendicular to its plane, the grains of sand will fly up some inches from it and be scattered in all directions.

b. In tangential vibrations, they only *glide* along close to the surface, and meet and settle on the nodal lines, and that, sometimes, with incredible swiftness. The reason why they retreat to the nodal lines is easily understood. The amplitude of the excursions of the vibrating molecules of the surface diminishes as we approach a nodal line. Hence a particle of sand anywhere situated, if thrown by an advancing vibration *towards* this line, will not be thrown quite so far back by the subsequent retreating vibration, because its *then* situation is one less agitated. Thus the motion of each particle of sand is one of alternate advances towards the node and recesses from it, but the advances are always greater than the recesses. In consequence, it *creeps* along the surface, and will not rest till it has attained the node.

147. The tangential vibrations of long flat rods or rulers of glass, as investigated by Savart, are extremely complicated, and offer most singular phenomena, some of which we shall now describe.

a. If we take a rectangular lamina of glass 27·56 inches long, 0·59 inches broad, and 0·06 inches thick, and holding it by the edges in the middle between the finger and thumb, with its flat face horizontal, strewed with sand, and, at the same time set it in longitudinal vibration, either by rubbing its under side near either end with a bit of wet cloth, by tapping it on the end with light blows, or by rubbing lengthwise a very small cylinder of glass, cemented on to its end in the middle of its breadth, and parallel to its length; in whatever way the vibration be communicated, we shall see the sand on its upper surface arrange itself in parallel lines, at right

angles to its longer dimension, *and always, in one or the other of the two systems*, represented in figs. 97 and 98.

Now it is very remarkable that, although the same one of these two systems will always be produced by the same plate of glass, yet among different plates of the above dimensions, *even though cut from the same sheet, side by side*, one will invariably exhibit one system, and the other the other, without any visible reason for the difference.

Moreover, in the system, fig. 97, the disposition of the nodal lines is unsymmetrical, one of them, *a*, being nearer to one end, and the closer pair, *f*, *f′*, not being situated in the middle; and this too is peculiar to the plate; for wherever it be rubbed, whichever end be struck, still the line *a* will always be formed nearest to the *same* extremity.

b. Now let the positions of the nodal lines be marked on the upper surface, and then let the plate be turned till the lower surface becomes the upper, and this being sanded, let the vibrations again be excited just as before. The nodal lines will now be formed quite differently, and will fall on the points just intermediate between those of the other surface; i. e. on the points of greatest excursion of its vibrating molecules. In a word if *n*, *n*, *n*, *n*, &c. in fig. 99 or 100, represent the places of the nodes on the one surface, then will *n′*, *n′*, &c. be those of the other.

Thus, *all the motions of one half the thickness of the lamina are exactly contrary to those of the corresponding points of the other half.* This property, indeed, is general, whatever be the material, length, breadth, or thickness of the lamina.

c. If, the other dimensions remaining, the thickness be increased, the *sound* will remain the *same*, *but the number of nodal lines will be less*. This fact alone is sufficient to prove an essential difference between the vibrating portions of such a plate, and the ventral segments of an organ-pipe harmonically subdivided.

d. If the breadth of a plate of the above length be greater than 0·6 inches, the nodal lines cease to be straight, and ranged across the plate at right angles to the sides. They pass into curves, and

when the breadth is increased to 1·57 inches, they assume the forms in figs. 101 and 102, the former representing the lines on the upper, the latter those on the under surface.

e. If the breadth be enlarged to 2·36 inches, the figures on the two faces will be as in figs. 103, 104.

f. If the dimensions be so varied as to convert the plate into a square, the nodal figures will assume the forms in figs. 105 and 106.

g. If the form of the plate pass into circular or triangular, the same mode of vibration (longitudinal-tangential) being preserved, still the opposite sides of the plate will present different nodal figures, as in 107, 108 and 109, 110.

148. In the longitudinal-tangential vibrations of cylindrical tubes or rods, there are two nodal lines which run spirally round the cylinder in opposite directions.

a. To examine the vibrations of cylinders, as sand will not lie on their convex surfaces, M. Savart employed the ingenious artifice used by Sauveur to exhibit the harmonic nodes of a vibrating string. For this purpose, the latter set astride on the string a small bit of paper cut into the form of an inverted V. But in this case it is found to answer better to encircle the vibrating cylinder with a narrow ring of paper, whose internal diameter is three or four times that of the cylinder, and which therefore hangs quite loosely on it.

b. If a cylinder of glass about 6½ feet long be encircled by several such *rings* or *riders*, and, being held horizontally by the middle, as lightly as possible, be rubbed in the direction of its length with a very wet cloth, it will yield a musical sound, and all the riders will glide rapidly along it to their nearest nodal points on the upper surface where they will rest. Now let all these points be marked, and then let the cylinder be turned so as to bring the opposite portion of its circumference uppermost and horizontal, and let the vibration be again excited in the same manner. Then we

shall remark the very same phenomenon as in rectangular plates, *viz.* that the nodal points on this edge correspond nearly to the middles of the intervals between those of the opposite one.

c. If the cylinder, instead of being turned at once half round, be turned only a little at a time, and always in the same direction, the riders will come to the points of rest constantly more and more towards one or the other end of the cylinder, according as it is turned to the right or the left; and if *the locus* of all the nodal points be traced by this means, it will be found to be a species of spiral line or screw, making one or more turns round the cylinder according to its length.

d. But there exists here a peculiarity, bearing an obvious relation to what we have observed already in the case of rectangular plates. The continuity of this spiral is interrupted near the middle of the cylinder, or rather it stops short at a point *n*, on one side of the central point, and recommences at *N*, a point equidistant on the other side; *but in a contrary direction*, so as to form on the two moieties of the length of the cylinder a right and a left-handed screw.

e. Again, these spirals are not equally inclined to the axis in all parts of their course. They consist of portions alternately much and little inclined, having points of maximum and minimum inclination alternately at every 90° of their course round the cylinder, as in fig. 111; thus dividing the cylinder into four quadrantal portions, which are related to each other in the same manner as the upper and under faces, and the right and left sides of the vibrating parallelopipeds examined in art. 147.

149. In the case of a hollow tube, the nodal lines of the internal surface consist of spirals, in all respects similar to those on the external surface; only that their coils run exactly along the intervals of those of the external one. So that in all cases, those points of the internal surface are most strongly agitated by the

vibrations which correspond to points at rest on the outer, and *vice versâ.*

a. The nodal lines of the internal surface ay be examined by strewing in it a little fine sand, provided its diameter be so large as not to drive all the sand into a crowded line along the bottom.

b. M. Savart has noticed a very curious phenomenon in this case. At the points of maximum inclination, the sand gathers itself up in a circular heap, and remains at perfect rest; but at those of minimum inclination, it forms a long ellipse, the borders of which keep constantly circulating in one direction; and if, instead of sand, a small globe of ivory or wax be put into the tube, it remains at these points, it is true, without shifting its *place*, but spins constantly in one direction round a vertical axis, so long as the vibration continues.

150. We have all along supposed that the state of vibration, into which the cylinder or tube is thrown, is that corresponding to the gravest tone it can yield by vibrations of the kind in question. M. Savart has examined its higher modes, and has pointed out other peculiarities. We will merely remark that, in these modes, the threads of the screw break off, and reverse their directions at the points of union of the several ventral segments.

CHAPTER II.

THE COMMUNICATION OF VIBRATIONS FROM ONE VIBRATING BODY TO ANOTHER.

151. We have already seen that a rod placed between two discs, one of which is set in vibration, becomes the means of communicating its vibrations to the other.

But it may be announced as a general proposition, that whenever a vibrating body is brought into intimate contact with another, it communicates to it its own vibrations, more or less effectually as their union is more perfect, and all the parts of the body thus set in vibration by communication are agitated by motions, not merely similar in their periods, but actually parallel in their directions to those of the original source of the motion.

This similarity of the communicated to the original vibrations was proved by the experiments of M. Savart, and is best illustrated by examples.

Example 1. Let *A*, fig. 112, be a long flat glass ruler, or rod, cemented with mastic to the edge of a large bell-glass, such as is used for the *harmonica*, or musical glasses, or a large hemispherical drinking-glass, perpendicular to its circumference. Let it be very lightly supported in a horizontal position on a bit of cork at *C*, and then let the bell-glass be set in vibration by a bow, at a point opposite the place where the rod meets it. It will vibrate *transversely*, i. e. the motions of its molecules will be perpendicular to its surface; and these motions will be communicated to the rod,

without any change in their direction, whose vibrations will be longitudinal-tangential, as will be rendered evident by strewing its surface with sand, when the nodal lines will be formed as in art. 147; and, if the apparatus be inverted, and the sand strewed on the under side of the rod, the nodal lines will be seen to correspond to the points of greatest excursion on the other side, as in that article.

In this combination the original tone of the bell-glass is altered, and the note produced differs both from that yielded by it, or by the glass rod vibrating alone. The two vibrate as a system together; and, what is singular, the sound of the glass is considerably reinforced by the combination.

Example 2. Let A', fig. 113, be a rectangular strip of glass, firmly cemented at right angles to another strip, A, across its breadth. Let the latter be lightly supported on two bits of cork, C, C', fastened to a wooden piece, B, so as just to touch A in the places of two of its nodes when vibrating transversely. Then if A be placed horizontally, and strewed with sand, and A' be set in longitudinal-tangential vibration, either by rubbing with a wet cloth, or by any other means, A will vibrate transversely, as will be known by the dancing of the sand, and its settling on the nodes, C, C'.

On the other hand, if A be held vertically, and agitated transversely by a bow, while A' is horizontal and strewed with sand, the latter will indicate longitudinal-tangential vibrations, both by the creeping of the sand, and by the difference of the nodal figures on its two faces.

Example 3. Let M, fig. 114, be a rectangular plate, mounted like A in the last example, but instead of carrying a simple plate A', let it carry a system of circular discs traversed by a lamina, as in the figure. Then, if the faces of these discs and of the lamina, M, be horizontally placed and strewed with sand, and the lamina, M, be set in longitudinal-tangential vibration, all the discs will be so too, and the sand will arrange itself in figures which, on every alternate disc, 1, 3, 5, &c. will be of one species, (such as at a for instance,) but on every other, 2, 4, 6, &c. will be of a different species, as b.

Communication of the Vibrations of a String to a Disc.

Now if the whole apparatus be inverted, so as to place the lamina, M, uppermost, and let the system of discs hang down, the *then* upper surfaces of the discs will exhibit the same system of nodal figures, but in the reverse order; i. e. the discs 1, 3, 5, &c. will give the figure b, and 2, 4, 6, &c. the figure a.

In this apparatus, if the connecting piece which traverses all the discs be examined, it will be found to vibrate transversely, while the discs and lamina, M, vibrate tangentially, and *vice versâ*.

Example 4. Let A, fig. 115, be a strong frame of wood of the form [, across the extreme edges of which is stretched a strong catgut, or other cord, and let LL' be a circular disc of glass, or metal, retained between the chord and back of the frame by the pressure of the former. Then, if the chord be set in vibration by a bow drawn transversely across it in one steady direction, the vibrations of the chord will all lie in the plane of the bow, and will be communicated in *the same* direction to the disc, which will execute tangential vibrations, all its molecules moving to and fro in lines parallel to the bow *through the whole extent of the disc.* This is easily verified by the direction in which sand strewed on it creeps.

Conceive the whole apparatus placed with the chord vertical, and projected on the plane of the horizon. If, as in fig. 115, a, FF^{v} be the projection of the bow, the surface of the disc will be marked with nodal lines parallel to it, the sand there being left, while that in the intermediate spaces creeps along to the edges, as marked by the arrows, and runs off.

If the projection of the bow, FF^{v}, be oblique to the line joining the points of support of the disc, as in fig. 115, b, the nodal line will be curved, as there shown, but the motion of the molecules of the sand going to form it will still be parallel to FF^{v}.

Finally, if the bow be drawn parallel to the line joining the points of support, as in fig. 115, c, the nodal line will be formed of two arcs making a cusp, but the same law of molecular motion will still hold good, as the arrows indicate.

Example 5. Let LL', fig. 116, be a rectangular lamina fastened at one end into a block, T, and at the other attached to a

cord, *c e*, stretched parallel to its length over a bridge, *e*, and put in vibration by a bow perpendicular to it, *FF'*.

Then, if the plane of the bow and string coincide with the plane of the surface of the lamina, as in fig. 117, *a*, the latter will execute tangential vibrations *across* its breadth, and will exhibit on its upper surface a single nodal line, $n\ n'\ n''$, as in fig. 117, but on its under none, all the sand being driven off.

Now incline the bow to the surface of the lamina, as represented in fig. 118, *a*, at an angle of about 20°, still keeping it perpendicular to the string, and the nodal line will assume the curvature represented in fig. 118.

If the bow be still more inclined, the curve breaks up, and at 45° of inclination, as in fig. 119, *a*, becomes changed into transverse and oblique lines, as in fig. 119; and it is now observed that the sand not only runs in the direction of the arrows, but also begins to leap, indicating an oblique vibration of the surface.

Lastly, when the bow is inclined 90° to the plane of the lamina, as in fig. 120, *a*, the vibration becomes altogether transverse, the nodal lines are similarly disposed on both sides of the plate, as in fig. 120, and the sand merely leaps up and down, till it is danced off the vibrating parts, without any tendency to creep.

152. The preceding proposition can also be extended to the vibrations of the air; and the motions of the aerial molecules in every part of a spherical wave, propagated from a vibrating body as a centre, instead of diverging like radii in all directions, so as to be always perpendicular to the surface of the wave, are all parallel to each other; in a word, they are disposed, not as in fig. 8, but as in fig. 7, agreeing with the remarks of art. 45.

And the same thing holds good, not only in air, but in liquids, as the experiments hereafter to be related satisfactorily demonstrate.

Nature of the Aerial Motions in a Sound-wave.

a. If a very thin membrane be stretched horizontally over the orifice of a circular bowl, as a drinking-cup, or *harmonica-glass*, (extremely thin paper wetted and glued to the edges, and then suffered to become tight by drying, answers very well,) and if fine sand be strewed on it, it becomes a most delicate detector of aerial vibrations.

b. Suppose now a circular disc of glass, held concentrically over it, with its plane parallel to that of the membrane, and set in transverse vibration, so as to form any of Chladni's acoustic figures, as for instance fig. 59. Then will this figure be imitated exactly by the sand on the membrane.

Now let the vibrating disc be shifted laterally, so as no longer to have its centre vertically over that of the membrane, but keeping its plane, as well as that of the membrane, horizontal. Still the figures marked out on the latter will be fac-similes of those on the disc, and that, whatever be the extent of lateral removal, till the vibrations become too much enfeebled by distance to have any effect at all.

c. But, in place of shifting the disc laterally, let its plane be inclined to the horizon. Immediately the figures on the membrane will change, though the vibrations of the disc remain unaltered, and the change will be the greater, the greater be the inclination of the plane of the disc to that of the membrane. And when the former plane is perpendicular to the horizon, the nodal figure on the membrane is found to be transformed into a system of straight lines, parallel to the common intersection of the two planes; and the particles of sand, instead of dancing, creep in opposite directions to meet in these lines. One of these always passes through the centre, and the whole system is analogous to what would be produced by attaching a cord to the centre of a disc, and, having stretched it very obliquely, setting it in vibration by a bow drawn parallel to the surface. In a word, the vibrations of the membrane are now tangential, and they preserve this character unchanged, however the disc be *now* shifted laterally, provided its plane be not turned from the vertical position.

If the disc be made to revolve about its vertical diameter, the nodal lines on the membrane will rotate, following exactly the motion of the disc.

153. The vibrations, communicated to a stretched membrane by the air, vary with the pitch, quality, and direction of the original vibrations, and also with the tension of the membrane.

a. If, *cæteris immutatis,* the pitch of the sound, whose vibrations, communicated through the air to the membrane, excite its motions, be altered, the membrane will still vibrate, *differing in this respect from a rigid lamina,* which will only vibrate by sympathy with sounds corresponding to its own subdivisions. The *membrane,* be it observed, will vibrate in sympathy with *any* sound, but every particular sound will mark out on it its own particular nodal figure, and as the pitch varies the figure varies. Thus if a slow air be played on a flute near it, each note will call up a particular form, which the next will efface, to establish its own.

b. Suppose the exciting cause be the vibration of a disc, or lamina of any form. If its mode of vibration be varied, so as to change its nodal figures, those on the membrane will vary ; and if the *same note* be produced by different subdivisions of different sized discs, the nodal figures on the membrane will be different.

c. The effect of a change in the direction of the primitive vibrations is clearly shown by the change of inclination of the disc in the experiment of the preceding article.

d. If the tension of the membrane be varied ever so little, most material changes will take place in the figures it exhibits. If paper be the substance employed, mere hygrometric changes affect it to such a degree, that, if moistened by breathing on it, and allowed to dry while the exciting sound is continued, the nodal forms will be in a constant state of fluctuation, and will not acquire permanence, till the paper is so far dried as the state of the surrounding atmosphere will permit.

Indeed, this fluctuation is so troublesome in experiments of this kind, that to avoid them it is necessary to coat the upper or exposed side of the paper with a thin film of varnish. Of all substances, which can be employed for the exhibition of these beautiful experiments, M. Savart observes, by far the best is such a varnished paper stretched on a frame and moistened on the under side. The moisture diminishes the cohesion of the fibres, and renders them nearly independent of each other, and indifferent to all impulses. As a proof of this, he observes, that he has frequently obtained, on a circular membrane of paper so prepared, a nodal figure composed of no fewer than twenty concentric annuli, which is far beyond what can be obtained in any other way.

154. A very important application of the properties of stretched membranes is, to employ such a one as an instrument for detecting the existence and exploring the extent and limits of contiguous and oppositely vibrating portions of masses of air.

For, since such a membrane is thrown into vibration by all aerial vibrations of a certain force, the fact of the existence, or not, of a vibratory motion in any point of the air, of a chamber, for instance, or a box, or large organ-pipe, may be ascertained by observing whether sand strewed on it is set in motion, and arranged in regular forms, on holding the membrane at that point.

Thus if an organ-pipe be made to sound with a constant force, and the *exploring membrane* be so far removed from it, that the membrane shall just cease to be visibly agitated, the force of the sound being increased by a quantity not sensible to the ear, the sand will recommence its motion.

Nay, if two such pipes placed close together, be made to *beat*, the membrane will be seen to be agitated at the coincidences, and at rest in the interferences of their vibrations.

b. Were the membrane to be entirely destitute of *tension*, its vibrations would exactly coincide with those of the air. But, as

it has always some tension or thickness, this modifies in a most complicated manner the effects of the direct aerial action.

c. In order to explore the actual state of the air in different parts of a vibrating mass of determinate figure, as to motion or rest, the sound should be excited and maintained by a constant cause at a high degree of intensity, especially if the mass of the air be large, as in a chamber or gallery; and to give the membrane the greatest possible sensibility, it ought to be stretched so as to be, naturally, in unison with the note sounded, so as to act as a receiver and condenser of the small aerial motions.

d. The greatest purity and intensity of the sounds, to be employed for this purpose, may be obtained by a *harmonica* glass, or the bell of a clock, maintained in vibration by a bow; and this may be still further augmented by adapting to it a resonant cavity, as, for instance, a large cylindrical vase of considerable diameter, closed at one end, and of such dimensions as separately to vibrate the same note. The tones thus produced, especially when large *harmonica* glasses are used, as M. Savart remarks, are of such intensity, that no ear can long support them, and at the same time, of such a rich and mellow quality, that all other musical sounds appear poor and harsh in comparison.

e. In order yet more to increase the sensibility of the membrane, the frame on which it is stretched should be fitted over the orifice of a similar resonant cavity. For convenience, and lest the tension of the membrane should vary by hygrometric changes, it is proper to have means of varying this at pleasure.

155. When the vibrations of the air in a chamber are examined, there are found to be nodal lines, which, in the case of rectangular chambers or galleries, wind spirally round the walls as in rectangular rods, and will even extend into the open air through a window.

These nodal lines are rendered evident to the ear by the diminution of sound; and there is also a re-

markable difference in the apparent direction of the sound in different positions of the ear with respect to the nodes.

a. Suppose that, being provided with the apparatus just described, we shut ourselves up in an apartment of regular figure, and free from furniture or projections from the walls, recesses, &c., and place one of our resonant cylinders with its axis horizontal, and the vibrating bell or glass opposite its orifice. In the direction of its axis place the membrane horizontally, with its proper frame and resonant cylinder below it, and strew the horizontal surface with sand.

If now, first, we place the membrane thus armed very near the source of the sound, it will vibrate with great force. As we withdraw it, (keeping it still in the line of the axis of the first resonant cylinder,) its vibrations will diminish gradually, and at length cease; after which, (still continuing to remove it along this line,) they will recommence and reach a maximum, at a point where their intensity is nearly equal to that close to the source of sound. Removing the membrane yet further, a new point of indifference is found, and so on till we reach the end of the chamber.

b. If we walk along the same line, keeping the ear in the plane of the horizontal axis of the resonant cylinder, we shall perceive the sounds to be much louder in the places where the vibrations of the membranes attain their maxima, than at the intermediate points where they are at their minima, and where the sounds would almost disappear, if it were not for the condensations and rarefactions of the air, which are there at their maxima.

c. At these latter a very curious phenomenon has been observed by M. Savart. When the auditor moves his head away from such a point, towards the *right*, (always supposing it to remain in the line of the axis above mentioned,) the sound will appear to come from the right, and if towards the left, it will seem to come from the left, whether the original source of sound be to the one or other side. This singular effect shows that the aerial molecules, on

either side of the point of indifference, are in opposite states of motion at any given instant. In making this experiment, the head should be so turned, that the axis of the resonant cylinder prolonged shall pass through both ears. Suppose, for instance, the sounding apparatus to be to the observer's left, and that his head be very near it. The sound will appear to enter at his left ear. As he removes farther away, so as to pass one of the nodes, it will seem as if the sound had changed sides, and now came from the right. When another node is passed, it will appear to have again shifted to the left, and so on.

d. But if we quit the axis of the cylinder, and carry an exploring membrane, such as already described, about the apartment, noting all the points where it vibrates most forcibly, allowing ourselves, as it were, to be led from spot to spot by its indications, we shall trace out in the air of the room a curve of double curvature, marking the maxima of the excursions of the aerial molecules.

If the experiment be made in a gallery, or passage, whose length is its principal dimension, this curve will be found to be a kind of spiral, creeping round the walls, floor, and ceiling, obliquely to the axis of the gallery, thus presenting a marked analogy to the disposition of the nodal lines in a long rod vibrating tangentially; which are also, it should be remarked, imitated, with modifications more or less complicated, in square or rectangular rods.

e. A still more remarkable effect was observed by M. Savart, in thus exploring the vibrations of the air in an apartment with an open window. The spiral disposition of the vibrating portions was found to be continued out of the window into the open air, the lines of greatest intensity running out in great convolutions, which seemed to grow wider, on receding from the window, and could be traced to a great distance from it.

156. The vibrations of a column of air contained in a pipe may also be examined by means of these membranes, and the positions of the nodes will be

found to be nearly as in art. 80, though they are somewhat affected by the embouchure, as might, in some degree, have been anticipated from the remarks of art. 85.

a. The vibrations of the air in an organ-pipe were explored by M. Savart, by lowering into the pipe, placed vertically with its upper end open, a thin membrane stretched on a light ring, and suspended by a fine silk thread, and strewed with sand. Thus ocular demonstration of the existence of its subdivision into distinct ventral segments was obtained, the sand remaining undisturbed when the membrane occupied precisely the place of a node.

b. By this means, too, the influence of the embouchure on the places of the nodes, a curious and delicate point in the theory of pipes may be subjected to exact examination. Thus, for instance, when the column of air in the pipe vibrates in the manner described in art. 79, (fig. 19,) having two half ventral segments, and one node in the middle, it is found that the node is only approximately so placed, being always, in fact, nearer to the embouchure than to the open end.

157. It is well known that, if we sing near the aperture of a wide-mouthed vessel, some one note (which is in unison with the air in the vessel) will be reinforced and augmented, and sometimes to a great degree. This is what is meant by the *resonance* of the mass of air contained in the cavity of the vessel, or as it may be termed, *the resonance of the cavity.*

a. This has been known from the earliest times. The ancients are said to have placed large brass jars under the seats of their immense theatres to reinforce (one does not well see how) the voices of the actors.

b. Any vessel or cavity may be made to resound by placing opposite its orifice a vibrating body, having a surface large enough

to cover the aperture, or at least to set a considerable portion of the aerial stratum adjacent to it in regular oscillation, and, at the same time, pitched in unison with the note which the cavity would of itself yield.

The experiment of the disked tuning-fork, in art. 93, is a case exactly in point. The pipe, which resounds in that experiment, may be pitched exactly in unison with it by its stopper, and in proportion as it departs from a perfect unison the resonance is feebler.

A series of disked tuning-forks, or vibrating steel springs, thus placed over the orifices of pipes carefully tuned, constitutes a very pretty musical instrument, capable of a fine swell and fall, according as the discs are brought nearer to, or further from, the orifices of the pipes, or inclined to their axes, and of remarkable purity and sweetness of tone.

A similar adaptation of resonant cavities to a series of harmonica glasses, fixed on a common revolving axis, has been recommended by M. Savart as the principle of a musical instrument, whose effect, should it be found to answer the expectations, which his description of the tones thus drawn forth is calculated to excite, would probably surpass that of all others yet invented. The cavities best adapted to this purpose are short cylinders of large diameters, with movable bottoms fitting by tight friction by which they may be tuned.

158. Such cavities may be regarded as short organ-pipes. When the diameter of a pipe is greatly increased in proportion to its length, so that it becomes a *box*, the law of the proportionality of the time of vibration to the length ceases to hold good, and the note yielded is flatter than that of a narrow pipe of equal length, and the more so the wider the pipe.

a. Thus M. Savart found that a cylinder of 4½ inches in length, and 5 in diameter, resounded in unison with a narrow pipe 6 inches long, making 1024 vibrations per second.

b. That sagacious experimenter has found, that cubical boxes *speak* with surprising promptitude and facility, and yield sounds extremely pure, and of a peculiar quality; on which account, and by reason of the little height in which they may be *packed*, he recommends them for organ-pipes. A cube of $4\frac{1}{2}$ inches in the side yields the same note as a pipe 10 or 11 inches long, and 2 or $2\frac{1}{2}$ inches in diameter.

They may be excited by an embouchure at one of their lower edges, precisely similar to that of an organ-pipe. But they will also speak if the embouchure be situated in the middle of the side.

c. M. Savart has also examined the vibrations of a great variety of different shaped pipes, boxes, or cavities.

159. There is yet another remarkable case of vibrations, communicated between the different members of a system, of which we have not yet spoken, though offering a good example of the verification of the general law of equality of period and parallelism of direction of the vibratory motions of all the molecules of a system laid down in art. 151. It is when vibrations are communicated through a liquid.

The following experiments of M. Savart will show the mode in which this is accomplished.

a. He took a cylindrical tinned iron vessel whose bottom was placed parallel to the horizon, and having cemented to its centre a glass rod, so as to hang perpendicularly down from it, he covered the bottom to the depth of about an inch and a half with water, on which was floated a thin disc of varnished wood, covered on its upper face with sand.

The apparatus thus prepared, he impressed on the glass rod a longitudino-tangential vibration, which of course became normal when communicated to the bottom of the vessel; and he observed the sand on the upper face of the disc to be agitated with normal

motions, and to assume nodal figures according to the laws of that species of motion.

To show more clearly the nature of the communication, he threw out the water, and supported the wooden disc by a small solid stem perpendicular to its surface, and the bottom of the vessel, and attached to the centres of both, when it was found that the disc was affected precisely in the same way as before.

b. On a vessel of water, whose rim is maintained in a state of normal vibration by a bow drawn perpendicularly across it at any point, let a thin rectangular lamina of wood be floated, having its length parallel to the bow, and its extremity opposite to the point of the circumference excited. The lamina will be seen (as usual by sand strewed on its upper face) to execute longitudino-tangential vibrations, and will be crossed by nodal lines at right angles to its length.

But if, instead of directing the axis or longer edges of the lamina perpendicularly towards the vibrating point of the side of the vessel, we incline it obliquely to the direction of the vibrations, still the sand on its upper face will continue to glide in the same direction as before, that is, parallel to the vibrations of the side of the vessel, so that, if the floating lamina be made to revolve slowly in a horizontal plane, the direction of the creeping motion of the sand on its surface will continually vary with respect to the position of its edges, though constant with regard to the sides of the vessel.

c. Not only are the vibrations thus faithfully transferred through the water to bodies floating on its surface, but even to such as are totally immersed in it.

The experiment is easily made by suspending in such a vessel as above described under the water, and not in contact with the sides or bottom, a disc of glass, by means of fine silk threads, and strewing sand on the surface of the water which sinks and spreads evenly on the disc. This will be observed to be agitated with very decided normal, or tangential motions, according as the former or the latter of the modes of excitement used in the preceding experiments is employed; and to arrange itself in nodal figures accordingly.

CHAPTER III.

THE FORMS AND STATES ASSUMED BY FLUIDS IN CONTACT WITH VIBRATING SURFACES.

160. WHEN a surface is vibrating in the air, or any other fluid, the fluid immediately in contact with the surface is formed into currents, which proceed from the nodal points to the points of greatest excursion.

As any particular part of the surface moves upwards in the course of its vibration, it propels the fluid and communicates a certain degree of force to it, perpendicular, or nearly so, to the vibrating surface; as it returns, in the course of its vibration, it recedes from the fluid so projected, and the latter consequently tends to return into the partial vacuum thus formed. But as, of two neighbouring portions of fluid, that over the part of the surface nearest to the point of greatest excursion has had more projectile force communicated to it than the other, because the part of the surface urging it was moving with greater velocity, and through a greater space, so it is in a more unfavorable condition for its immediate return, and the other, i. e. the portion next to it towards the nodal line, presses into its place.

This effect is still further favored, because the portion of fluid, thus displaced, is urged from similar causes at the same moment into the place left vacant by the fluid still nearer the point of greatest excursion; so that each time the surface recedes from the fluid, an advance of the fluid immediately above it is made from the quiescent to the vibrating parts of the surface.

161. The existence of these currents may be rendered evident by strewing the vibrating surface with

Collection of Fine Powder at the Points of Greatest Excursion.

some substance sufficiently light to be carried forward by the fluid to the points of greatest excursion, in opposition to the tendency which there otherwise is to the nodal lines, as in art. 127.

a. M. Chladni first observed that shavings from the hair of the exciting violin bow did not proceed to the nodal lines, but were gathered together on those parts of the vibrating plate the most violently agitated. Thus, when a square plate of glass held horizontally was nipped above and below at the centre, and made to vibrate by the application of a violin bow to the middle of one edge, so as to produce the lowest possible sound, sand sprinkled on the plate assumed the form of a diagonal cross; but the light shavings were gathered together at those parts towards the middle of the four portions, where the vibrations were most powerful and the excursions of the plate the greatest.

Many other substances exhibited the same appearance. Lycopodium, which was used as a light powder by Oersted, produced the effect very well; and these motions must not be confounded with those described in art. 146, arising from tangential vibrations.

These effects were also observed by M. Savart, and particularly investigated by him in circular, rectangular, triangular, and other plates; and in rods, rings, and membranes. Figs. 121–136 exhibit some of the arrangements of the powder which he obtained.

b. M. Savart, however, attributed this disposition of the powder to a wrong cause, namely, to a *secondary* mode of vibration; and Mr. Faraday was the first to propose the true explanation, and establish it beyond a doubt by a series of most ingenious experiments.

c. The motions of the powder, as observed by Mr. Faraday, are as follows. Let the plate just mentioned, which may be three or four inches square, be nipped and held in a horizontal position by a pair of pincers of the proper form, and terminated, at the [illegible] touching the glass, by two pieces of cork; let lycopodium

powder be sprinkled over the plate, and a violin bow be drawn downwards against the middle of one edge, so as to produce a clear full tone. Immediately the powder on those four parts of the plate towards the four edges will be agitated, whilst that toward the two diagonal cross lines will remain nearly or quite at rest.

On repeating the application of the bow several times, a little of the loose powder, especially that in small masses, will collect upon the diagonal lines, and thus show the position of the nodal lines.

Most of the powder which remains upon the plate will, however, be collected in four parcels; one placed near to each edge of the plate, and evidently towards the place of greatest agitation. Whilst the plate is vibrating (and consequently sounding) strongly, these parcels will each form a rather diffuse cloud, moving rapidly within itself; but as the vibration diminishes, these clouds will first considerably contract in bulk, and then settle down into four groups, each consisting of one, two, or more hemispherical parcels, which are in an extraordinary condition; for the powder of each parcel continues to rise up at the centre, and flow down on every side to the bottom, where it enters the mass to ascend at the centre again, until the plate has nearly ceased to vibrate. The form of these heaps, and the involved motion they acquire, are, however, no part of the phenomenon under consideration at present, but will be hereafter explained.

If the plate be made to vibrate strongly, the heaps are immediately broken up, being thrown into the air, and form clouds, which settle down as before; but if the plate be made to vibrate in a smaller degree, by a more moderate application of the bow, the little hemispherical parcels are thrown into commotion without being sensibly separated from the plate, and often slowly travel towards the nodal lines.

When one or more of them have thus receded from the place over which the clouds are always formed, and a powerful application of the bow is made sufficient to raise the clouds, it will be seen that these heaps rapidly diminish, the particles of which they are composed being swept away from them, and passing back in a current over the glass to the cloud under formation, which ultimately settles as before into the same four groups of heaps.

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d. So powerful are the currents of air, that, when the vibrations are energetic, the plate may be inclined 5°, 6°, or 8° to the horizon, and yet the gathering clouds retain their places. As the vibrations diminish in force, the little heaps formed from the clouds will descend the inclined plane ; but on strengthening the vibrations, the heaps will melt away, their particles will ascend the plane and pass again to the cloud. This will take place when the inclination of the plate is so great, that neither sand nor filings can rest on the nodal lines.

e. A piece of gold-leaf being laid upon the plate, so that it does not overlap the edge, as in fig. 137, the current of air towards the points of greatest excursion is beautifully shown ; for, by its force, the air creeps in under the gold-leaf on all sides, and raises it up into the form of a blister ; that part of the gold-leaf corresponding to the centre of locality of the cloud, when light powder was used, being frequently a sixteenth or twelfth of an inch from the glass.

Lycopodium, or other fine powder, sprinkled round the edge of the gold-leaf, is carried in by the entering air, and accumulated underneath.

f. When fine powder is placed on the edge of another glass plate, or upon a book, or block of wood, and the edge of the vibrating plate brought as nearly as possible to the edge of the former, (fig. 138,) part of the powder is always driven on to the vibrating plate, and collected in the usual place; showing that, in the midst of all the agitation of the air in the neighbourhood of the two edges, there is a current towards the point of greatest excursion even from bodies not themselves vibrating.

g. In the exhausted receiver of an air-pump the phenomena do not occur as in air; for, as the force of the currents is excessively weakened, the light powders assume the part of heavier grains in the air.

To observe this difference in the phenomena, Mr. Faraday made use of a circular stretched membrane, so excited that the powder collected in air at the centre, as in fig. 121. Upon exhausting the

receiver until the barometric gauge was at twenty-eight inches, the powder, instead of collecting at the centre, passed across the membrane towards one side which was a little lower than the other. It passed by the middle just as it did over any other part; and when the force of the vibration was much increased, although the powder was more agitated at the middle than elsewhere, it did not collect there, but went towards the edges or quiescent parts.

Upon allowing the air to enter until the barometer stood at twenty-six inches, and repeating the experiments, the effect was nearly the same. When the vibrations were very strong, there were faint appearances of a cloud, consisting of the very finest particles, collecting at the centre; but no sensible accumulation of the powder took place.

At twenty-four inches of the barometer the accumulation at the centre began to appear, and there was a sensible, though very slight, effect visible of the return of the powder from the edges.

At twenty-two inches these effects were stronger; and when the barometer was at twenty inches, the currents of air within the receiver had force enough to cause the collection of the principal part of the powder at the centre of vibration.

Upon again restoring the exhaustion to twenty-eight inches, all the effects were reproduced as at first, and the powder again proceeded to the lower or quiescent parts of the membrane.

h. In denser media than air, as in water, for instance, heavier powder, as sand and filings, performs the part of light powders in air, and is carried from the quiescent to the vibrating parts.

162. The currents may be obstructed by pieces of card, and directed to points different from those of greatest excursion.

a. If a piece of card, about an inch long and a quarter of an inch wide, is fixed by a little soft cement on the face of the square plate, used in the preceding article, near one edge, the plate held as before at the middle, fine powder strewed upon it, and the bow applied at the middle of another edge; the powder immediately advances

close to the card, and the place of the cloud is much nearer the edge than before. Fig. 139 represents the arrangement; the diagonal lines being those which sand would have formed, the line at the top, *a*, representing the place of the card, and the × to the right, the place where the bow was applied; *b* represents the place of the cloud when no card is present, and it does not proceed to the point of greatest excursion at the very edge of the plate, on account of the manner in which the air is there agitated.

b. If pieces of card are fixed on the glass in the three angular forms represented in fig. 140, upon vibrating the plate the fine powder always goes into the angle, notwithstanding its difference of position in the three experiments, but perfectly in accordance with the idea of currents intercepted more or less by the card.

c. When two pieces of card are fixed on the plate, as in fig. 141, *a*, the powder proceeds into the angle, but not to the edge of the glass, remaining about $\frac{1}{8}$th of an inch from it; but on closing up that opening, as at *b*, the powder goes quite up into the corner. Upon fixing two pieces of card on the plate, as at *c*, the powder between them collects in the middle, very nearly as if no card had been present; but that on the outside of the cards gathers close up against them, being able to proceed so far in its way to the middle, but no further.

d. When a long glass plate is supported by bridges or strings at the two nodal lines, represented in fig. 142, and made to vibrate, the powder collects in three divisions. That between the nodal lines does not proceed at once into a line equidistant from the nodal lines and parallel to them, but advances from the edges of the plate towards the middle by paths, which are a little curved and oblique to the edges where they occur near the nodal lines, but are almost perpendicular to it elsewhere; and the powder gradually forms a line along the middle of the plate; it is only by continuing the experiment for some time, that it gathers up into a heap or cloud equidistant from the nodal line.

But upon fixing card walls upon the plate, as in fig. 143, the course of the powder within the cards is directly parallel to them and to the edge, instead of being perpendicular, and also directly towards the point of greatest excursion.

To prove that it is not as a weight that the card acts, but as an obstacle to the currents of air formed, it is bent flat down outwards, without being moved from its place, and then the powder resumes the courses it took when without the cards.

The powder sprinkled over the extremities of the plate proceeds towards places equidistant from the sides and near the ends, as at *a*, fig. 144; but on cementing a piece of paper to the edge, so as to form a wall about one quarter or one third of an inch high, *b*, the powder immediately moves up to it, and retains this new place.

In a longer narrow plate, similarly arranged, the powder can be made to pass to either edge, or to the middle, according as paper interceptors to the currents of air are applied.

163. The air, carried forward by the currents to the points of greatest excursion, rises from the vibrating surface at these points, proceeds to a greater or less distance from the surface, and then returns to the nodal lines, forming currents in opposite directions to the first, a little above them, and blending more or less with them.

a. Mr. Faraday endeavoured, in various ways, to make the extent of this system of currents visible. In the experiment already referred to, where gold-leaf was placed over the point of greatest excursion, the upward current at the most powerful part was able to raise the leaf about one tenth of an inch from the plate. The clouds formed at the points of greatest excursion are always higher and larger, as the vibrations are stronger, and may frequently be seen rising up in the middle and flowing over towards the sides.

b. In the receiver of an air-pump, if the powder collected at the middle of the circular stretched membrane is observed, as to the height to which it is forced upwards by the vibrations; and then the receiver being exhausted, if the height to which the powder is thrown by similar vibrations is again observed; in the latter case it is nothing like so great as in the former, the height not being two-thirds, and barely one-half, the first height. Were the powder thrown up by mere propulsion, it should rise far higher in vacuo than in air; but the reverse takes place, because that in air the current has force enough to carry the fine particles up to a height, far beyond what the mere blow which they receive from the vibrating membrane can effect.

c. When the circular stretched membrane is made to vibrate, and a large glass tube, as a cylindrical lamp-glass is brought near to its centre; the most striking proofs are obtained of the existence of carrying currents by the effects upon the light powder, which flies rapidly under the edge, and tends to collect towards the axis of the tube; it may even be diverted somewhat from its course towards the point of greatest excursion. A piece of upright paper, held with its edge equally near, does not produce the same effect; but immediately that it is rolled into a tube, it does.

When the glass chimney is suspended very carefully, and at but a small distance from the membrane, the powder often collects at the edge, and revolves there; a complicated action between the currents and the space under the thickness of the glass taking place, but still tending to show the influence of the air in arranging and disposing the powder.

d. Let a sheet of drawing-paper be stretched tightly over a frame, so as to form a tense elastic surface nearly three feet by two in extent. Upon placing this in a horizontal position, throwing a spoonful of lycopodium upon it, and striking it smartly below with the fingers, the phenomena of collection at the point of greatest excursion, and of moving heaps, can be obtained upon a magnificent scale. When the lycopodium is uniformly spread over the surface, and any part of the paper slightly tapped by the hand, the

lycopodium at any place chosen can be drawn together merely by holding the lamp-glass over it. It will be unnecessary to enter into the detail of the various actions combining to produce these effects; it is sufficiently evident, from the mode in which they may be varied, that they depend upon currents of air.

e. A very interesting set of effects occur, when the circular stretched membrane is vibrated under plates; the powder collects with much greater rapidity than without the plate; and instead of forming the semi-globular moving heaps, it forms linear arrangements, all concentric to the centre of the membrane.

When the vibrations are strong, these assume a revolving motion, rolling towards the centre at the lower part in contact with the membrane, and from it at the upper part nearest the glass; thus illustrating in the clearest manner the double currents caged up between the glass and the membrane.

f. Sometimes, when the plate is held down very close and tight, and the vibrations are few and large, the powder is all blown out at the edge; for then the whole arrangement acts as a bellows; and as the entering air travels with much less velocity than the expelled air, and as the forces of the currents are as the squares of the velocity, the issuing air carries the powder more forcibly than the air which passes in, and finally throws it out.

164. When a surface, vibrating *transversely*, is covered with a layer of liquid, that liquid, if of sufficient tenacity, is determined from the quiescent to the vibrating parts, producing accumulation at the latter places; but this accumulation is limited, so that, if purposely rendered too great by gravity or other means, it will quickly be diminished by the vibrations, until the depth of fluid at any one part has a certain and constant relation to the velocity there, and to the depth elsewhere.

Vibrating Surfaces Covered with a Layer of Liquid.

a. Let a round plate, held horizontally by the middle, be covered with oil over the upper surface, so as to be flooded, except at ×, fig. 145, and the bow applied at × to produce strong vibration. The oil is immediately accumulated at *a*, *b*, and *c*, forming fluid lenses there, rendered evident by their magnifying power, when print is looked at through them. The accumulations are also visible on putting a sheet of white paper beneath, in consequence of the color of the oil being deeper at the accumulations than elsewhere; and they are also rendered beautifully evident by making the experiment in sunshine, or by putting a candle beneath the plate, and placing a screen on the opposite side to receive the images formed at the focal distance.

When the vibration of the plate ceases, the oil gradually flows back, until of uniform depth. On renewing the vibration, the accumulations are re-formed.

b. To remove every doubt of the fluid passing from the quiescent to the agitated parts, points of excursion may be used, nearly surrounded by nodal lines. A square plate, fig. 146, being held at *c*, and the bow applied at ×, gives with sand nodal lines resembling those in the figure. Then clearing off the sand, putting oil in its place, and producing the same mode of vibration as before, the oil accumulates at *a* and *b*, forming two heaps or lenses as in the former experiment.

c. If the round plate is again used, and the oil placed on its under surface, the oil will hang in drops upon it, which may be made to stand at the points *a*, *b*, and *c*. On applying the bow at ×, and causing the plate to vibrate, the drops instantly disappear, the oil being gathered up and expanding laterally over the plate. But this lateral diffusion is soon limited; for lenses are formed at the points of greatest excursion, just as when the oil was upon the upper surface, and, as far as can be ascertained by general examination, of the same form and power. On stopping the vibration, the oil gathers again into hanging drops; and on renewing it, it is again disposed in the lens-like accumulations.

d. The reasoning of art. 160 may be applied to the present case, but the upper currents of art. 163 will, if the mobility of the liquid is very great, carry it so rapidly back to the nodal lines, that the accumulation cannot be observed; and this effect is assisted by the tendency which the particles of the fluid, like those of sand, have to the nodal lines, and which the tenacity of the liquid may not be sufficient to overcome.

e. Let a square plate be covered with water, and vibrated as in the former experiments; all endeavours to ascertain whether accumulation occurs at the points of greatest excursion, either by direct observation, or the reflection from its surface of right-lined figures, or by looking through the parts, as through a lens, at small print and other objects, fail.

As, however, when the plate is strongly vibrated, peculiar crispations form on the water at the points of greatest excursion, and prevent any possible decision as to accumulation, it is only when these are absent, and the vibration weak, and the accumulation therefore small, that any satisfactory result can be expected; but as even then no appearance is perceived, it seems that the force of gravity, combined with the mobility of the fluid, is sufficient to restore the uniform condition of the layer of water, after the bow is withdrawn, and before the eye has time to observe the convexity expected.

f. To remove in part the effect of gravity, or rather to make it coincide with, instead of oppose, the convexity, the under surface of the plate may be moistened instead of the upper, and by inclining the plate a little, the water made to hang in drops at *a* or *b* or *c*, fig. 147, at pleasure. On causing the plate to vibrate, the drops of water, like those of oil, are gathered up and diffused over the plate.

On stopping the vibration, it again accumulates in hanging drops, which instantly disappear as before, on causing the plate to vibrate, the force of gravity being entirely overpowered by the superior forces excited by the vibrating plate. Still no visible evidence of convexity at the points of greatest excursion are obtained,

and the water appears rather to be urged from the vibrating parts than to them.

165. The peculiar manner, in which the fine powder upon a vibrating surface is accumulated into little heaps, either hemispherical or merely rounded, and larger or smaller in size, has already been described, in art. 161 *c*, as well also as the singular motion which they possess, as long as the plate continues in vibration. These heaps form on any part of the surface which is in a vibratory state, and not merely under the clouds produced at the centres of vibration, although the particles of the clouds always settle into similar heaps.

They have a tendency, as heaps, to proceed to the nodal lines, but are often swept away by the currents of art. 160. When on a place of rest, they do not acquire the involving motion.

When two or more are near together, or touch, they will frequently coalesce and form but one heap, which quickly acquires a rounded outline. When in their most perfect and final form, they are always round.

a. The moving heaps formed by lycopodium on large stretched drawing-paper, are on so large a scale as to be very proper for critical examination. The phenomena can be exhibited also even by dry sand on such a membrane, the sand being in large quantity, and the vibrations slow. When the surface is thickly covered by sand from a sieve, and the paper tapped with the finger, the manner in which the sand draws up into moving heaps is very beautiful.

b. When a single heap is examined, which is conveniently done by holding a vibrating tuning-fork in a horizontal position, and

dropping some lycopodium upon it, it will be seen that the particles of the heap rise up at the centre, overflow, fall down upon all sides, and disappear at the bottom, apparently proceeding inwards; and this evolving and involving motion continues until the vibrations have become very weak.

c. If a square glass plate be vibrated as in former experiments, its surface having been covered with sand enough to hide the plate, and water enough to moisten and flow over the sand, the sand will draw together in heaps, and these will exhibit the peculiar and characteristic motion of the particles in a very striking manner.

166. The aggregation and motion of these heaps, either in air or other fluids, is a very simple consequence of the mechanical impulse, communicated to them by the joint action of the vibrating surface and the surrounding medium.

a. That the medium in which the experiment is made has an important influence, is shown by the circumstance of heavy particles, such as filings, exhibiting all these peculiarities when they are placed upon surfaces vibrating in water; the heaps being even higher at the centre than a heap of equal diameter formed of light powder in the air. In water, too, they are formed indifferently upon any part of the plate or membrane which is in a vibratory state. They do not tend to the nodal lines; but that is merely from the great force of the currents formed in water as already described, and the power with which they urge obstacles to the place of greatest vibration.

b. When, in the course of a vibration, the part of a plate under a heap rises, it communicates a propelling force upwards to that heap, mingled as it is with the fluid in which the experiment is made, greater than that communicated to the surrounding fluid, because of the superior specific gravity of the former; upon receding from the heap, therefore, in performing the other half of its vibration, the plate forms a partial vacuum, into which the fluid,

round the heap, enters with more readiness than the heap itself; and, as it enters, carries in the powder at the bottom edge of the heap with it. This action is repeated at every vibration; and as they occur in such rapid succession that the eye cannot distinguish them, the central part of the heap is continually progressing upwards; and as the powder thus accumulates above, whilst the base is continually lessened by what is swept in underneath, the particles necessarily fall over and roll down on every side.

c. Although this statement is made upon the relation of the heap, as a mass, to the fluid surrounding it, yet it will be seen at once that the same relation exists between any two parts of the heap at different distances from the centre; for the one nearest the centre will be propelled upward with the greatest force, and the other will be in the most favorable state for occupying the partial vacuum left by the receding plate.

d. This view of the effect will immediately account for all the appearances: the circular form, the fusion together of two or more heaps, their involving motion, and their existence upon any vibrating part of the plate. The manner in which the neighbouring particles would be absorbed by the heaps is also evident; and as to their first formation, the slightest irregularities in the powder or surface would determine a commencement, which would then instantly favor the increase.

e. It is quite true, that if the powder were coherent, that force alone would tend to produce the same effect, but only in a very feeble degree. This is sufficiently shown by the experiments made in the exhausted receiver. When the barometer of the air-pump is at twenty-eight inches, that in the air being about 29·2 inches, the heaps, or rather parcels, form very beautifully over the whole surface of the membrane; but they are very flat and extensive compared with the heaps in air, and the involving motion is very weak. As the air is admitted, the vibration being continued, the heaps rise in height, contract in diameter, and move more rapidly. Again, in the experiments with filings and sand in water, no cohe-

sive action could assist in producing the effect; it must be entirely due to the manner in which the particles are mechanically urged in a medium of less density than themselves.

f. The conversion of these round heaps into linear concentric involving parcels, in the experiment already described when the membrane was covered by a plate of glass, is a necessary consequence of the arrangements there made, and tends to show how influential the action of the air, or other including medium, is in all these phenomena.

167. When a vibrating surface is covered with a layer of any fluid, the fluid usually presents a beautifully crispated appearance in the neighbourhood of the points of greatest excursion.

a. This appearance was observed by Oersted, Wheatstone, Weber, and probably others, but was first analyzed and its true theory given by Mr. Faraday. It is easily produced upon a square glass plate, held horizontally, as in most of the preceding experiments, covered with sufficient water on the upper surface to flow freely from side to side when inclined, and made to vibrate strongly by a bow applied to one edge, ×, fig. 148, in the usual way. Crispations appear on the surface of the water, first at the points of greatest excursion, and extend more or less toward the nodal lines, as the vibrations are stronger or weaker. The crispation presents the appearance of small conoidal elevations of equal lateral extent, usually arranged rectangularly with extreme regularity; permanent in appearance, so long as a certain degree of vibration is sustained; increasing and diminishing in height, with increased or diminished vibration; but not affected in their *lateral extent* by such variations, though the whole crispated surface is enlarged or diminished at those times.

If the plate be vibrated so as to produce a different note, the crispations still appear at the point of greatest excursion, but are smaller for a high note, larger for a low note. The same note

produced on different sized plates, by different modes of vibration, appears to produce crispations of the same dimension, other circumstances being the same.

These appearances are beautifully seen when ink diluted with its bulk of water is used on the plate.

b. It is necessary for examination both to prolong and enlarge the effect, and the following are advantageous modes of producing it used by Mr. Faraday. Plates of crown glass, from eighteen to twenty-two inches long, and three or four inches wide, were supported each by two triangular pieces of wood, acting as bridges, as in fig. 142, and made to vibrate by a small glass rod or tube resting perpendicularly at the middle, over which the moist fingers are passed. By sprinkling dry sand on the plates, and shifting the bridges, the nodal lines were found (usually about one fifth of the whole length from each end), and their places marked by a file or diamond. Then clearing away the sand, putting water or ink upon the plate, and applying the rod or fingers, it was easy to produce the crispations, and sustain them undisturbed, and with equal intensity for any length of time.

c. By making a broad mark, or raising a little ledge of bee's wax, or a mixture of bee's wax and turpentine, it is easy to confine the pool of water, to the middle part of the plate, fig. 149, where, of course, the crispations are most powerfully produced. Such a barrier is often useful to separate the wet and dry parts of the glass, especially when a violin bow is used as the exciter.

d. Deal laths, two, three, or four feet long, one inch and a half wide, and three eighths or more of an inch in thickness, were also used instead of the glass plates. These can be made to vibrate by the fingers and wet rod, and by either shifting the bridges or changing the lath an almost unlimited change of isochronous vibrations, from that producing a high note to those in which not more than five or six occur in a second, can be obtained. The crispations are formed upon a glass plate, attached to the middle of the lath by two or three little pellets of soft cement.

Obtained in this way, the appearances are very beautiful, and the facilities very great. A glass plate, from four to eight inches square, can be covered uniformly with crispations of the utmost regularity; for, by attaching the plate with a little method, and at points equidistant from the centre of the bar, it is easy to make every part travel with the same velocity, and in that respect differ from, and surpass, the bar which sustains it. The conoidal heaps constituting the crispation can be so enlarged by slowness of vibration, that three or four may occupy a linear inch. The glass plate can be removed, and another of different form or substance, and with other fluids, as mercury, &c., substituted in an instant.

The exciting glass rod need not necessarily rest upon the middle of the bar or plate, but may be applied with equal effect at some distance from it. Long laths may be made to subdivide in their mode of vibration, according as the rod is applied to different places, and the pressure given by the exciting moist fingers is varied; with each change of this kind an immediate change of the crispation is observed.

e. This form of apparatus was enlarged by Mr. Faraday, until a board eighteen feet long was used, the layer of water being now three-fourths of an inch in depth, and twenty-eight inches by twenty inches in extent. The sides of the cistern were very much inclined, so that the water should gradually diminish in depth, and thus reflected waves be prevented. The vibrations were so slow as to be produced by the direct application of the hand, and the heaps were each from an inch to two inches in extent. Though of this magnitude, they were identical in their nature with those forming crispations on so small a scale, as to appear merely like a dullness on the surface of the water.

168. The proportion of water requires a general adjustment to the extent of the vibrations, the crispations being produced more readily and beautifully, when there is a certain quantity, than when there is less.

For small crispations, the water should flow upon the surface freely. Large crispations require more water than small ones. Too much water sometimes interferes with the beauty of the appearance, but the crispation is not incompatible with much fluid, for the depth may amount to eight, ten, or twelve inches, and is probably unlimited.

169. The crispations are equally produced upon the under with the upper surface of vibrating plates.

When the lower surface is moistened, and the bow applied, the drops which hang down by the force of gravity are rippled; but being immediately gathered up, as in art. 164, *c*, a certain definite layer is produced, which is beautifully rippled, or crispated, at the point of greatest excursion.

170. Most liquids, if not all, may be used to produce these crispations, but some with peculiar advantages.

a. Alcohol, oil of terpentine, white of egg, ink, and milk produce them. White of egg, notwithstanding its viscosity, shows them readily and beautifully. Ink has great advantages, because, from its color and opacity, the form of the surface is seen undisturbed by any reflection from the glass beneath; its appearance in sunshine is exceedingly beautiful. When diluted ink is used for large crispations, upon tin plate, or over white paper, or mercury, the different degrees of color, or translucency corresponding to different depths of the fluid, give important information relative to the true nature of the phenomena. Milk is, for its opacity, of similar advantage, especially when a light is placed beneath, and being more viscid than water is better for large arrangements, because it produces less splashing. Oil does not show small crispations readily, but when warmed (by which its liquidity is increased) it produces them freely. Cold oil will also produce large crispations, and for very large ones would probably be better than water, because of its cohesion.

b. The difference between oil and white of egg is remarkable; for the latter, from common observation, would appear to be a thicker fluid than oil; but the qualities of cohesion differ in the two, the apparent thickness of white of egg depending upon an elastic power, (probably due to an approach to structure,) which tends to restore its particles to their first position, and coexisting with great freedom to move through small spaces, whilst that of oil is due to a real difficulty in removing the particles one by another.

It is possible that the power of assuming, more or less readily, the crispated state may be a useful, and even important, indication of the internal constitution of different fluids.

c. With mercury the crispations are formed with great facility, and of extreme beauty, when a piece of amalgamated tin or copper plate, being fixed on a lath, is flooded with the fluid metal, and then vibrated. A film quickly covers the metal, and then the appearances are not so regular as at first; but on removing the film by a piece of paper, their regularity and beauty are restored. It is more convenient to cover the mercury with a little very dilute acetic or nitric acid; for then the crispations may be produced and maintained for any length of time, with a surface of perfect brilliancy.

d. When a layer of ink is put over the mercury, the acid of the ink removes all film, and the summits of the metallic heaps, by diminishing the thickness of the ink over them, become more or less visible, producing the appearance of pearls of equal size beautifully arranged in a black medium.

When mercury covered with a film of dilute acid is vibrated in the sunshine, and the light reflected from its surface on a screen, it forms a very beautiful and regular image; but the screen requires to be placed very near to the metal, because of the short focal lengths of the depressions on the mercurial surface.

171. When the crispations are observed, well formed with vibrations so slow as to produce three or four elevations in a linear inch, they are seen to be

conoidal heaps rounded above, and apparently passing into each other below by a curvature in the opposite direction.

When arranged regularly, each is surrounded by eight others, so that in the reflection of a single light nine images are sent from each elevation to the eye.

The lateral dimensions of the heap remain constant, notwithstanding considerable variations in the force of vibration. But a variation in the depth of fluid affects their number; so that with less fluid the heaps are smaller, and with more fluid larger, though the sound, and, therefore, the number, of vibrations in a given period remains the same.

The crispation on the long plate of glass, in fig. 149, always ultimately assumes a rectangular arrangement, i. e. the heaps are equidistant, and in rows parallel, or at right angles, to each other. The rows usually form angles of 45° to the sides of the plate at the commencement; but if the vibrations are continued, the whole system usually wheels round through 45°, until the rows coincide with the edges of the plate.

a. With this plate the appearances are usually in the following order, the pool of water being quadrangular, or nearly so, and the exciting rod resting in the middle of it.

Ring-like linear heaps concentric to the exciting rod first form to the number of six or seven; these may be retained by a moderated state of vibration, and produce intervals which, measured across the diameter of the rings, are to the number of ten in three inches, with a certain constant depth of water. By increasing the force of vibration the altitude of these elevations increases, but not their lateral dimension, and then linear heaps form across these circles and the plate, and parallel to the bridges, having an evident relation to the manner in which the whole plate vibrates. These,

which, like all other of these phenomena, are strongest at the part most strongly vibrating, soon break up the circles, and are themselves broken up, producing independent heaps, which at first are irregular and changeable, but soon become uniform, and produce the quadrangular order; first at angles of 45° to the edges of the plate, but gradually moving round until parallel to them. So the arrangement continues, unless the force be so violent as to break it up altogether; if the vibratory force be gradually diminished, then the heaps as gradually fall, but without returning through the order in which they were produced. The lines, fig. 150, may serve to indicate the course of the phenomena. When perfectly formed, the intervals between the heaps are the same as were those between the rings, the number of heaps being ten in three inches, with the same depth of water as that which produced the rings. But the number can be reduced to eight, or increased to eleven and a half, in the three inches, by a change in no other condition than the depth of fluid.

b. Another form of heaps occasionally occurs, but always passing ultimately into those described. These heaps are grouped in an arrangement still very nearly rectangular, and at angles of 45° to the sides of the plate; but are contracted in one direction, and elongated in the other; these directions being parallel to the sides and ends of the plate. If the marks in fig. 151 be supposed to represent the tops of the heaps, an idea of the whole will be obtained. Three inches along these heaps in the direction of *AB* include eight heaps, but across them in the direction of *AC* include fifteen nearly. These numbers are therefore the relation of length to breadth. But along the lines of the quadrilateral arrangement, *AD* or *BC*, three inches include eleven heaps, which, notwithstanding the difference in form, is the same number that was produced by the same plate, with the same depths of water, when the heaps were round; therefore an equal number of heaps exist in the same area in both cases; and the departure from perfect rectangular arrangement, and also the ratio of 1 : 2, is probably due to some slight influence of the sides of the plate.

When mercury covered with a film of very dilute nitric acid is vibrated, the rectangular arrangement is constantly obtained. When vibrated under dilute ink, it is still more beautifully seen and distinguished. When the tin plate sustaining the mercury is square, and the whole surface is covered with crispations, the lines of the rectangular arrangement are always at angles of 45° to its edges.

172. When sand is sprinkled uniformly over a plate, on which large water crispations are produced, i. e. four, five, or six in the inch, it gives some very important indications. It immediately becomes arranged under the water, and with a little method may be made to yield very regular forms. It is always removed from under the heaps, passing to the parts between them, and frequently producing therefore the form, fig. 152, of great regularity. As the sand figure remains when the vibration has ceased, it allows of the determination of position, the measurement of intervals, &c. very conveniently.

Very often the lines of sand are not continuous, but separated with extreme regularity into portions, as represented fig. 153. The portions of these lines are sometimes, with little sand on the plate, very small, fig. 154; and when more sand is present, they are thickened occasionally, fig. 155; then assuming the appearance of heaps, arranged in straight lines at angles of 45° to the lines regulating the position of the water-heaps which form them, and just double in number to the latter.

Sometimes the sand, instead of being deficient at the intersecting angle, will accumulate there only, fig. 156; and at other times will accumulate there principally, but still show the original form by a few connecting particles, fig. 157.

When the heaps are of the form, fig. 151, the sand is still washed from under them; it does not, however, assume lines parallel to the rectangular arrangement of the heaps, but is arranged as in fig. 158.

When only the circular linear heaps are produced, the sand assumes similar circular forms, concentric and alternating with the water elevations.

173. On strewing a little lycopodium over the water, for the purpose of gaining information relative to what occurs at the surface during the crispation, it moves about over the fluid in every possible direction, whilst the crispations exist of the utmost steadiness beneath. The same thing occurs with pieces of cork on very large crispations. But when much lycopodium is put on, so that the particles retain each other in a steady position, then it forms lines parallel to the arrangement of the heaps, the powder being displaced from the parts over the heaps, and taking up an arrangement perpendicularly over the sand beneath.

As the lycopodium forms float on the water, they are easily disturbed, and in no respect approach, as to beauty and utility, to the forms produced by the sand; but lycopodium may be used with smaller crispations than sand.

174. The crispations are much influenced by various circumstances. They tend to commence at the place of greatest vibration; but if the quantity of fluid is too little there, and more abundant elsewhere, they will often commence at the latter place first. Their final arrangement is also much affected by the form of the plates, or of the pool of water on which they occur.

a. When the plates or pools are rectangular, and all parts vibrate with equal velocity, the lines of heaps are at angles of 45° to the edges. But when semi-circular and other plates are used, the

arrangement, though quadrangular, is unsteady, often breaking up and starting by pieces into different and changing positions.

b. When mercury is used, the film formed on it after a few moments has great power, according to the manner in which it is puckered, of modifying the general arrangement of new crispations.

c. When a circular plate, supported by cork feet attached where a single nodal line would occur, is covered with water, and vibrated by a rod resting upon the middle, the crispations extend from the middle towards the nodal line. These are sometimes arranged rectangularly, but have no steadiness of position, and change continually. At other times, the heaps appear as if hexagonal, and are arranged hexagonally, but these also shift continually. This and many other experiments show that the direction and nature of the vibrations of the plate (i. e. of the lines of equal or varying vibrating force) have a powerful influence over the regularity and final arrangement of the crispations.

175. The beautiful appearance, exhibited when the crispations are produced in sunshine, or examined by a strong concentrated artificial light, has been already referred to. When the reflected image from any one heap is examined, it is found not to be stationary, as would happen if the heap was permanent and at rest; nor yet to form a vertical line, as would occur if the heap were permanent, but travelled to and fro with the vibrating plate; but it moves so as to re-enter upon its course, forming an endless figure, like those produced by Dr. Young's piano-forte wires, in art. 67, or Wheatstone's kaleidophone, varying with the position of the light and the observer, but constant for any particular position and velocity of vibration.

Upon placing the light and the eye in positions nearly perpendicular to the general surface of the fluid, so as to avoid the direct

influence of the motion of vibration, still the luminous, linear, endless figure is produced, extending more or less in different directions, according to the relation of the light and eye to the crispated surface, and occasionally corresponding in its extent one way to the width of the heap, i. e. to the distance between the summit of one heap and its neighbours, but never exceeding it. The figure produced by one heap is accurately repeated by all the heaps when the vibrating force of the plate is equal and the arrangement regular.

176. The heaps which compose the crispations are not permanent elevations, like the cones of lycopodium powder, the fluid rising at the centre and descending down the inclined sides; but they are raised and destroyed with each vibration of the plate.

Moreover, the heaps do not all exist at once, but form two sets of equal number and arrangement, fig. 159, never existing together, but alternating with, and being resolved into each other, and by their rapidity of recurrence, giving the appearance of simultaneous and even permanent existence.

Each heap recurs, or is reformed in two complete vibrations of the sustaining surface. But as there are two sets of heaps, a set occurs for each vibration. The maximum and minimum of height for the heaps appear to be, alternately, almost immediately after the supporting plate has begun to descend in one complete vibration.

a. On producing a water crispation, having four or five heaps in a linear inch, placing a candle beneath, and a screen of French tracing paper above it, the phenomena are very beautiful, and such as agree with this description. By placing the screen at different distances, it can be adapted to the focal length due to the curva-

ture at different parts of the surface of fluid, so that by observing the luminous figure produced and its transitions, as the screen is moved nearer or further, the general form of the surface can be deduced. Each heap with a certain distance of screen gives a star of light ⊕, fig. 160, which twinkles, i. e. appears and disappears alternately, as the heap rises and falls. At the corners × equidistant from these, fainter starred lights appear; and by putting the screen nearer to or further from the surface, lines of light, in two or even four directions, appear intersecting the luminous centres, apparently permanent, whilst circumstances remain unchaned. These effects can be magnified to almost any scale.

b. When heaps of similar magnitude are produced, with diluted ink on glass, and white paper, or an illuminated screen looked at through them, a chequered appearance is observed. In one position, lines of a certain intensity separate the heaps from each other, but the square places, representing the heaps look generally lighter. In another position, when but little reflected light comes from the surface of the heaps, their places can be perceived as dark from the greater depth of ink there.

By care, another position can be found in which the whole surface looks like an alternate arrangement of light and dark chequers, fig. 161, not steady, but with a quivering motion, which further attention can trace as due to a rapid alternation in which the light spaces become dark, and the dark light, simultaneously. When, instead of glass, a bright tin plate is used under the diluted ink, the chequered spaces and their alternations can be seen still more beautifully.

c. It was in consequence of these effects that very large arrangements were made by Mr. Faraday, giving heaps that were two inches and a half wide each: and then it was evident, by ordinary inspection, that the heaps were not stationary, but rose and fell; and also that there were two sets regularly and alternately arranged, the one set rising as the other descended.

Sand gave no indications of arrangement with these large heaps; but when some coarse saw-dust was soaked, so as to sink in water,

and then distributed in the fluid, its motions were beautifully illustrative of the whole philosophy of the phenomena. It was immediately washed away from under the rising and falling heaps, and collected in the places equidistant between these spots, as the sand did in the former experiments, and by its vibratory motion to and fro, it showed distinctly how the water oscillated from one heap towards another, as the heaps sunk and rose.

When milk was used instead of water, for these large arrangements, in a dark room, and a candle was placed beneath, the appearances also were very beautiful, resembling in character those in fig. 161.

d. When regular crispations have heen sustained for a short time on mercury, on which a certain degree of film has been allowed to form; on examining the film afterwards in one light, lines can be seen on it, coinciding with the intervals of the heaps in one direction; in another light, lines coinciding with the other direction come into sight, whilst the first disappear; and in a third light both sets of lines can be seen cutting out the square places where the heaps had existed; in these spaces the film is minutely wrinkled and bagged, as if it had there been distended; at the lines it is only a little wrinkled, giving the appearance of texture; and at the crossing of the lines themselves, it is quite free from mark and fully distended. All these are natural consequences, if the film be considered as a flexible, but inelastic, envelope formed over the whole surface whilst the heaps are rising and falling.

177. The mode of action, by which the heaps are formed, is analogous in some points to that by which the currents and the involving heaps already described are produced, and all the phenomena observed and described, the quadrangular arrangement of the heaps, their successive destruction and re-formation, the increase of their number, with that of the vibrations, the necessity of a certain depth of fluid, the permanence of

the lateral extension of the heaps, the arrangements of the sand and lycopodium, can readily be explained on mechanical principles.

a. The plate in rising tends to lift the overlying fluid, and in falling to recede from it; and the force which it is competent to communicate to the fluid, can, in consequence of the physical qualities of the fluid, be transferred from particle to particle in any direction. The heaps are at their maximum elevation just after the plate begins to recede from them; before it has completed its motion downwards, the pressure of the atmosphere, and that part of the force of the plate which through cohesion is communicated to them, has acted, and by the time the plate has begun to return, it meets them endowed with momentum in the opposite direction, in consequence of which they do not rise as heaps, but expand laterally, all the forces in action combining to raise a similar set of heaps, at exactly intermediate distances, which attain their maximum height just after the plate again begins to recede; these therefore, undergo a similar process of demolition, being resolved into exact repetitions of the first heaps. Thus the two sets oscillate with each vibration of the plate, and the action is sustained so long as the plate moves with a certain degree of force; much of that force being occupied in sustaining this oscillation of the fluid against the resistance offered by the cohesion of the fluid, the air, the friction on the plate, and other causes.

b. A natural reason appears for the quadrangular and right-angled arrangement which is assumed, when the crispation is most perfect. The hexagon, the square, and the equilateral triangle are the only regular figures which can fill an area perfectly. The square and triangle are the only figures which can allow of one half alternating symmetrically with the other, in conformity with what takes place between the two reciprocating sets of heaps, figs. 159 and 162; and of these two the boundary lines between squares are of shorter extent than those between equilateral triangles of equal area. It is evident, therefore, that one of these two will be finally assumed, and that that will be the square arrange-

ment; because then, the fluid will offer the least resistance in its undulations to the motions of the plate, or will pass most readily to those positions into which the forces, it receives from the plate, conspire to impel it.

c. The fluid may, then, be considered as a pendulum, vibrating to and fro under a given impulse; the various circumstances of specific gravity, cohesion, friction, intensity of vibrating force, &c. determining the extent of oscillation, or, what is the same, the number of heaps in a given interval; the distance between two heaps, or the lateral extent of a heap, being that passed over by a wave on the surface of the fluid during two vibrations of the plate. Hence, when the number of vibrations in a given time is increased, the heaps are more numerous.

The necessity of a certain depth of fluid is evident, and also the reason why, by varying the depth, the lateral extent of the heaps is changed. The arrangement of the sand and lycopodium, by the crispations, and the occurrence of the latter at centres of vibration, and only upon surfaces vibrating normally, are all evident consequences.

d. As to the origin or determination of crispations, no difficulty can arise; the smallest possible difference in almost any circumstance, at any one part, would, whilst the plate is vibrating, cause an elevation or depression in the fluid there; the smallest atom of dust falling on the surface, or the smallest elevation in the plate, or the smallest particle in the fluid, of different specific gravity from the liquid itself, might produce this first effect; this would, by each vibration of the plate, be increased in amount, and also by each vibration extended by the breadth of a heap, in at least four directions; so that in less than a second a large surface would be affected, even under the improbable supposition that only one point should at first be disturbed.

e. The circular linear heaps produced on long or circular plates by feeble vibration are explicable upon the same principles, account being at the same time taken of the arrangement and proportion of vibrating force in the various parts of the plates.

178. The heaps which constitute the crispations here described are not to be confounded with what are called *stationary undulations*, although of the same form, quality, and motion of their parts; but which are produced by an entirely different cause, being the result of different progressing undulations formed partly of reflections from the sides of the vessel containing the fluid, and depending upon the form of the vessel.

a. If the mercury in a small circular basin is tapped at the middle, stationary undulations, resembling the ring-like heaps, are obtained, which arise from two sets of progressing and opposed undulations, one of which proceeds from the centre to the circumference of the surface, and the other is the reflection proceeding from the circumference to the centre.

b. If a rectangular frame be made to beat at equal intervals of time on mercury or water, heaps like those of the crispations, arranged quadrangularly at angles of 45° to the frame, are produced, which result from the four sets of undulations propagated from the sides of the frame. Thus the undulations from the opposite sides of the frame, being opposed, produce lines parallel to each other, so that two sets of such lines are formed equidistant and perpendicular to each other, and which, being superposed like the vibrations on rectangular plates, in art. 129, *e*, produce heaps arranged at angles of 45° to the frame.

c. The heaps of crispations, on the contrary, have no dependence on progressive undulations originating laterally, as many of the phenomena described prove. Thus, when the edges are bevelled or covered with cloth, or wet saw-dust, so that waves reaching the side shall be destroyed, or when the limits of the water or plates are round or irregular, still the heaps are produced, and their arrangement square. When the round plate is used, regular crispations are still produced, though, as the water extends

over the nodal line, and is there perfectly undisturbed, no progressing and opposed undulations can originate to produce them. Vellum stretched over a ring, and rendered concave by the pressure of the exciting rod, produces the same effect.

d. When a plate of tin, rendered very slightly concave, is attached to a lath, so as to have equality of vibratory motion in all its parts, and a little dilute alkali (which will wet the surface), put into it, the crispations form in the middle, but cease towards the sides, where, though well wetted, there is not depth enough of water, and from whence also no waves can be reflected to produce stationary undulations in the ordinary manner.

e. When a similar arrangement is made with mercury on a concave tin plate, the effects are still more beautiful and convincing. The centre portion is covered with one regular group of quadrangular crispations; at some distance from the centre, and where the mercury is less in depth, these pass into concentric, ring-like heaps, of which there are a great many; and outside of these there is a part wet with mercury, but with too little fluid to give either lines or heaps. Here there can be no reflected waves; or, if that were thought possible, those waves could not form both the circular rings and the square crispation. When this plate is vibrated, the mercury spreads in all directions up the side, a natural consequence of the production of powerful oscillations at the middle, which will extend their force laterally, but quite against their being due to the opposition and crossing of waves originating at the sides.

179. When a vertical plate is vibrated transversely at the horizontal surface of a fluid, crispations are formed which consist of linear heaps arranged perpendicularly to the plate.

a. On arranging the long plates, fig. 142, vertically, so that the lower extremity dips about one third of an inch into water, fig. 163, and causing it to vibrate by applying the rod at ×, or by tapping the plate with the finger, undulations of a peculiar character are

observed; those passing from the plate towards the sides of the basin are scarcely visible though the plate vibrates strongly, but in place of such, appear others, in the production of which the mechanical force of the vibrating plate exerted upon the fluid is principally employed. These are apparently permanent elevations, at regular intervals, strongest at the plate, projecting directly out from it over the surface of the water, like the teeth of a coarse comb gradually diminishing in height, and extending half or three quarters of an inch in length. These vary in commencing at the glass, or having intervening ridges, or in height, or in length, or in number, or in breaking up into violently agitated pimples and drops, &c. according as the plate dips more or less into the water, or vibrates more or less violently, or subdivides whilst vibrating into parts, or changes in other circumstances.

But when the plate, (sixteen or seventeen inches long,) dips about one sixth of an inch, then four of these linear heaps occupy as nearly as possible the same space, as four heaps formed with the same plate in the former way and accompanied with the same sound.

b. By fixing a wooden lath perpendicularly downwards in a vice, plates of any size or form can be attached to its lower end and immersed more or less in water; and by varying the immersion of the plate, or the length of the lath, or the place against which the exciting rod is applied, the vibrations can be varied in rapidity to any extent.

On using a piece of board at the extremity of the lath, eight inches long, and three inches deep, with pieces of tin plate four inches by five, fixed on at the ends in a perpendicular position to prevent lateral disturbance at those parts, very regular and beautiful ridges are obtained of any desired width, fig. 164. These ridges, as before, form only on the wood, and are parallel to the direction of its vibration. They occur on each side of the vibrating plane with equal regularity, force, and magnitude, but seem to have no connexion, for sometimes they correspond in position, and at other times not; the one set shifting a little, without the others being displaced.

c. It can now be observed that the ridges on either side the vibrating plane consist of two alternating sets; the one set rising as the other falls. For each fro and to motion of the plane, or one complete vibration, one of the sets appears, so that in two complete vibrations, the cycle of changes is complete. Pieces of cork and lycopodium powder show that there is no important current setting in the direction of the ridges; towards the heads of the ridges, pieces of cork oscillate from one ridge towards its neighbour, and back again. The lycopodium sometimes seems to move on the ridges from the wood, and between them to it; but the motion is irregular, and there is no general current outwards or inwards. There is not so much disturbance as amongst the heaps of art. 167.

d. A very simple arrangement exhibits these ripples beautifully. If an oval or circular pan, fifteen or eighteen inches in diameter, be filled with water, and a piece of lath twelve or fifteen inches long be held in it, edge upwards, so as to bear against the sides of the pan, as supporting points, and cut the surface of the water, then on being vibrated horizontally by the glass rod, and wet finger, the phenomenon immediately appears with ripples an inch or more in length. When the upper edge of the lath is an inch below the surface, the ripples can be produced. When the vessel has a glass bottom, the luminous figures produced by a light beneath, and a screen above, are very beautiful. Glass, metal, and other plates, can thus be easily experimented with.

e. These crispations are perfectly analogous, as to cause, arrangement, and action, with the heaps and crispations already explained in art. 177, and display the same vibratory motion in directions perpendicular to the force applied, by which the water can most readily accommodate itself to rapid, regular, and alternating changes in bulk in the immediate neighbourhood of the oscillating parts.

180. Similar crispations to those just described are produced, when a substance is made to vibrate in contact with and perpendicularly to the surface of a fluid, or indeed in any other direction.

a. Let a lath be fixed horizontally in a vice by one end, so that the other can vibrate vertically; let a cork be cemented to the under surface of the free end, and a basin of water placed beneath with its surface just touching the cork, as in fig. 165. On vibrating the lath by means of a glass rod and the fingers, a beautiful and regular star of ridges, two, three, or even four inches in length, is formed round the cork. These ridges are more or less numerous according to the number of vibrations, &c. As the water is raised, and more of the cylinder immersed, the ridges diminish in strength, and at last disappear; when it just touches the surface, they are most powerfully developed. This is a necessary consequence of the dependence of the ridges upon the portion of water, which is vertically displaced, and restored at each vibration. When this portion is at or near the surface, the ridges are freely formed in its immediate vicinity; when at a greater depth, (being always at the bottom of the cork,) the displacement is diffused over a larger mass and surface, each particle moves through less space, and with less velocity, and, consequently, the vibrations must be stronger, or the ridges be weaker, or disappear altogether.

The refraction of a light through this star produces a very beautiful figure on a screen.

b. A heavy tuning-fork vibrating, but not too strongly, if placed with the end of one limb either vertical, inclined, or in any other position, just touching the surface of water, ink, milk, &c. shows the effect very well for a moment. It also shows the ridges on mercury, but the motion and resistance of so dense a body quickly bring the fork to rest. It forms ridges in hot oil, but not in cold oil.

With cold oil, a very inclined fork produces a curious pump-like action, throwing up four streams, easily explained when witnessed, but not so closely connected with the present phenomena, as to require more notice here.

181. When a large glass of water is made to sound by passing the wet finger round the edges, the glass

divides into four vibrating parts, opposite to which the crispations are strongest, and there are four nodal points at equal distances from each other.

If the vessel is a large glass jar, and soft sounds are produced, the surface of the water exhibits the ridges at the centres of vibration; as the sound is rendered louder, these extend all round the glass, and at last break up at the centres of vibration into irregular crispations; but both the ridges and crispations are effects of the kind already described, and require no further explanation.

182. Ridges, several inches long, are sometimes produced on the surface of smooth shallow water, by the force of the wind, differing from waves of the ordinary kind, in being accurately parallel to the course of the wind, and resembling the crispations just described.

These ridges may be observed, during a strong steady wind, on a smooth, flat, sandy shore, with enough water on it, either from the receding tide, or from the shingles above, to cover it thoroughly, but not to form waves, in a place where the wind is not broken by pits or stones. They are of uniform width, whatever the extent of surface, varying in width only as the force of the wind, and the depth of the stratum of water varies. They may be seen at the windward side of the pools on the sand, but break up as soon as waves appear. If the waves be quelled by putting some oil on the water to windward, these ripples, then, appear on those parts.

They are often seen, but so confused that their nature could not be gathered from such observations, on the pavements, roads, and roofs, when sudden gusts of wind occur with rain.

As they could not be caused by the wind exerted in a manner similar to that by which ordinary waves are produced, they are probably due to the water, acquiring an oscillatory condition similar to those described, probably influenced in some way by the elastic nature of the air itself.

They are strong enough to arrange the sand beneath where ordinary waves had not been powerful enough to give form to the surface.

183. All the phenomena of crispation, as yet described, are such as take place at the *surfaces* of those fluids, in common language considered as inelastic, and in which the elasticity, they possess, performs no necessary part; nor is it possible that they could be produced within their mass. But on extending the reasoning, it does not seem at all improbable that analogous effects should take place in gases and vapor, their elasticity supplying that condition necessary for vibration, which in liquids is found in an abrupt termination of the mass by an unconfined surface.

a. If this be so, then a plate vibrating in the atmosphere may have the air immediately in contact with it separated into numerous portions, forming two alternating sets like the heaps on the surface of water; the one denser, and the other rarer, than the ordinary atmosphere; these sets alternating with each other by their alternate expansion and condensation with each vibration of the plate.

b. With the hope of discovering some effect of this kind, Mr. Faraday made use of a flat circular tin plate, having a raised edge of tin, three quarters of an inch high, fixed on all round, and the plate was then attached to a lath, a little lycopodium put on to it, and vibrated powerfully, so that the powder should form a mere cloud in the air, which, in consequence of the raised edge and the equal velocity of all parts of the plate, had no tendency to collect.

Immediately it was seen that in place of a uniform cloud it had a misty honeycomb appearance, the whole being in a quivering condition; and on exerting the attention to perceive waves, as it were, travelling across the cloud in opposite directions, they could be most distinctly traced. This is exactly the appearance that

would be produced by a dusty atmosphere, lying upon the surface of a plate, and divided into a number of alternate portions, rapidly expanding and contracting simultaneously.

c. But the spaces were very many times too small to represent the interval through which the air, by its elasticity, would vibrate laterally once for two vibrations of the plate, in analogy with the phenomena of liquids. This would form a strong objection to this being an effect of the kind proposed, were it not possible that the air may have vibrated in subdivisions, like a string or a long column of air, which subdivision would probably be produced, if by no other circumstance, by reflection from the raised edges of the plate, which must take place when the edge is nearly at a point of greatest condensation and rarefaction.

d. If the atmosphere vibrates laterally in the manner supposed, the effect is probably not limited to the immediate vicinity of the plate, but extends to some distance. Thus the vertical plates of fig. 164 produced ripples on the water, five or six inches long; whilst the waves parallel to the vibrating plate were hardly sensible; and something analogous to this might take place in the atmosphere. If so, it would seem likely that these vibrations, occurring conjointly with those producing sound, would have an important influence upon its production and qualities, upon its apparent direction and many other of its phenomena.

CHAPTER IV.

THE EAR.

184. THE auditory apparatus, represented by fig. 166, is very complicated; most of its parts are extremely small, being contained in the small cavity of a very hard bony projection, which extends on each side of the head into the interior of the skull, and is part of the temporal bone. It may be divided into three parts, the outer, the middle, and the inner ear.

185. The *outer ear* consists of a wide, conch-shaped opening, *a b*, which contracts into a narrow pipe, *c d*, defended from the entry of dusts and insects, by hairs and a viscous exudation which is slowly secreted.

The form of the opening of the ear in many animals, as the hare, the horse, &c. is that of a horn, or of the ear-trumpet used by the deaf, and is calculated to reflect the vibrations of air which strike upon it, and to augment the intensity of the sound.

The human ear, on the contrary, expands at its opening into a broad rim, which is rather fitted, by its position and the inequalities of its surface, to receive the vibrations which are nearly perpendicular to it, and transmit them through its substance to the bones of the head, and thence to the internal ear, than to reflect sound into the ear. It is, however, of but little use, and its loss does not much weaken the power of hearing; but the rest of the external ear is shaped like a horn, and undoubtedly serves to augment the sounds which it transmits.

186. The *middle ear* consists of *the cavity of the tympanum*, *h*; *the membrane of the tympanum*, *g*; two openings called *foramens* nearly opposite this membrane, one *round*, the other *oval*, the *Eustachian tube*, and *four little bones* which traverse the cavity of the tympanum, from its membrane to that of the *foramen ovale*.

a. The *cavity of the tympanum* is chiefly designed as a reservoir for the air, which is communicated to it through the long canal of the *Eustachian tube*, and which is thereby maintained at a constant state of temperature and moisture, all direct communication with the outer air being cut off by means of the *membrane of the tympanum*. The object of maintaining this constant temperature and moisture is to preserve the membranes of the two *foramens* from those great changes in their modes of vibration, mentioned in art. 153 *d*, which *stretched* membranes always undergo during such changes, and which would affect our perception of the quality, though not of the pitch, of sounds.

The resonance of the cavity of the tympanum is also probably of some use in reinforcing the sounds transmitted to it.

b. The *membrane of the tympanum*, which closes the auditory passage and excludes the outer air, is not necessary for hearing; and it would be a great impediment to the admission of sound, if it were not so thin and of so little tension, that, as in art. 154 *b*, its vibrations almost exactly coincide with those of the air. Moreover it is moistened on the inner side, like the most sensitive membranes used by M. Savart, art. 153 *d*; it is placed over the mouth of the resonant cavity of the tympanum, which still increases its sensibility, as in art. 154 *e*; and the air in this cavity is not so closely confined as to impede its motions, but can expand when compressed through the Eustachian tube, like the air in a drum through the hole in the side of the drum.

c. Were it not for the supply of air through the Eustachian tube, the air in the cavity of the tympanum would soon be absorbed, and

the vibrations of the tympanum would be but feebly transmitted to the internal ear through the bones of the cavity. Thus the obstruction of this tube is sometimes the cause of deafness.

d. The four little bones represented in fig. 167, which extend across the cavity of the tympanum, consist of the *hammer*, *SC*, which rests with its smaller end in contact with the middle of the membrane of the tympanum; the *anvil*, *BP*, which, by a little *round bone*, *P*, communicates with the last, *V*, called the *stirrup*, and attached to the membrane of the *foramen ovale*. The design of these bones is to restrain the membranes of the *foramens* from vibrating too powerfully under the action of intense sounds, and thus injuring the auditory nerve. They effect this purpose by the pressure of the *stirrup* on the membrane of the oval *foramen*, which is communicated to the round *foramen* through the liquid of the internal ear, and by which the tension of these membranes is increased, and, therefore, the extent of their vibrations diminished.

187. The *inner ear*, or *the labyrinth*, fig. 168, is a hard bony cavity, consisting of several smaller cavities, all communicating together, called the *vestibule*, *A*; *three semi-circular canals*, *BBB*, and the *cochlea*, *C*; it is filled with a limpid albuminous fluid, called the *perilymphe* or the *liquid of Cotugno*, within which floats *the membranous labyrinth* also filled with a limpid fluid, like the white of egg, and containing branches of the *auditory nerves* and several *calcareous concretions*.

a. The labyrinth is a mystery, the parts of which are well enough described by the anatomist, but their respective uses are altogether unknown, and hardly a plausible conjecture has been offered respecting them. The *vestibule* appears to be the essential organ of hearing, as it is never wanting even in the most imperfect ears of animals, the lowest in the scale of being; and the additions of the *semicircular canals* and the *cochlea* seem designed to give the power of discerning the *qualities* of different sounds, but

how they effect this purpose is, as yet, unexplained. The vestibule is immediately connected with the *cavity of the tympanum* by means of the *foramen ovale.*

b. The *cochlea* is a most singular organ of a spiral form, with its cavity divided into two parts by a partition, half bony, half membranous; it is filled with the perilymphe, contains a branch of the auditory nerve, has a free communication with the vestibule, and is separated from the cavity of the tympanum only by the round *foramen.*

c. The two *fluids* contained in the labyrinth are of such cohesive power, that they easily retain the membranes and *calcareous concretions* floating within them in a constant position; and yet they are of such elastic power, and the mobility of their parts is so perfect, that, like white of egg, art. 170 *b*, they readily admit of vibrations. Their elasticity is not, however, so great that, like air, art. 183, they would admit of crispations within their substance near the membranes of the foramens, and thus disturb the perception of the quality of sounds.

The importance of the *perilymphe* is shown by the fact, that, when the membrane of either *foramen* is injured so that this liquid can flow out, the hearing is destroyed.

d. The *membranous labyrinth* does not extend through the whole of the bony cavity, but is limited to the *vestibule* and the *semicircular canals.* In the vestibule it has the form of several bags, or *sacks,* united together, and it extends in the form of a tube throughout the semicircular canals, bulging out in each of them, near one end, into the form of a swelling. The bags of the vestibule and the swellings in the tubes contain, each of them, several calcareous concretions, which are attached to the auditory nerves, and seem to perform some important office in the communication of vibrations to it; as loose bones, which supply their place in inferior ears, are always found even in the most defective ears, those which consist only of a vestibule, containing the two fluids, and a single membranous sack.

CHAPTER V.

THE VOICE.

188. In all animals, without exception, (unless, perhaps, the grasshopper with its chirp, or the cricket be such,) the sounds of the voice are produced by a *wind instrument*, the column of air contained in the mouth, throat, and anterior part of the windpipe being set in vibration, by the issue of a stream of air from the lungs through a membranous slit in a kind of valve placed in the throat.' In man and in quadrupeds this organ is single ; but in birds it is double, a valve of this kind being placed at the opening of each of the two great branches, into which the trachea first divides itself as it enters the lungs, just before they unite in one common windpipe.

Almost every animal has a voice, or cry, peculiar to itself; the voice being most perfect, and varied, in man and in birds, which, however, differ extremely in the degree in which they possess this important gift. In quadrupeds, it is limited to a few uncouth screams, bellowings, and other noises, perfectly unmusical in their character, while in many birds it assumes the form of musical notes, of great richness and power, or even of articulate speech. In the human species alone, and that only in some rare instances, we find the power of imitating with the voice every imaginable kind of noise, with a perfect resemblance, and of uttering musical tones of a sweetness and delicacy attainable by no instrument.

189. The organs of the voice, in man, consist of the *thorax*, the *trachea*, the *larynx*, the *pharynx*, the *mouth*, and the *nose*, with their appendages, all of which necessarily concur in the production of almost every sound; and this remarkable concurrence, which renders it very difficult to discover in every case the peculiar office of each organ, is produced by a most complicated and perfectly adjusted system of nerves.

190. The *thorax*, by the aid of the diaphragm, and the 24 intercostal muscles acting on the lungs within, and alternately compressing and dilating them, performs the office of a bellows.

Now it is important that the force of the blast of a wind instrument should be adapted to its dimensions, and also to the different notes which it sounds; and this adaptation is effected in the church-organ by proportioning with mathematical accuracy the dimensions of all the pipes, which sound the different notes, to the same pair of bellows. But the motions of the human chest are so admirably adjusted to the changes of the vocal organs in sounding the different notes, that the church-organ could not be made to approach the precision of adjustment in the human organs, were there as many pairs of bellows as there are pipes, and each adjusted by a weight or spring, to accommodate the pressure of air to the dimensions of the pipes.

191. The *trachea*, and all that portion of the windpipe, which extends from the larynx to the lungs, may be considered as the *porte-vent*, or tube, which conveys the air from the lungs to the musical organs; and it probably has some influence upon the quality and pitch of the voice, but more upon the vowel sounds, as in art. 195.

The Larynx.

192. The *larynx* may be considered as the musical organ of the voice, being that upon which the pitch chiefly depends.

a. The *larynx*, fig. 169, is a short tube, situated in the throat, at the head of the trachea, and just behind the *pomum Adami*, composed of five elastic cartilages, of which the uppermost is called the *epiglottis*, whose office is to open and shut, like a valve, the orifice of the larynx, during the process of swallowing. It is lined or *hung* within, with a fine mucous membrane which forms, near its middle, two great lateral folds, called the *vocal chords*, or the *inferior ligaments of the glottis;* and these chords can be stretched, or the *chink* between them, which resembles a button-hole, contracted by the action of the ligaments and muscles connected with them. A little above the vocal chords are two other similar folds of the membrane, called the *upper ligaments* of the larynx. The space shut in by the membrane between these ligaments is called *the glottis;* a front vertical section of which is represented in fig. 170, *a a* being the *inferior* ligaments, and *b b* the superior ligaments.

b. The *glottis* has been compared by M. Savart to the *bird-call* used by hunters. This instrument consists of a short cylindrical tube, closed at each end by a round plate, through which is made a small hole. A current of air, being blown through the holes, carries along with it part of the air contained in the cavity, and produces a more and more rarefied state of air within the instrument, until the outer air by its superior pressure either forces back the whole of the issuing stream, or part of it near the edge of the hole, and thus condenses the air within, until a reaction is produced, followed by another rarefaction, and so on.

Now M. Savart has found that the sounds, which are thus produced, vary in pitch with the force of the blast, and with changes in the dimensions of the instrument, and in the size of the apertures; and they are still more modified, if the air passes through a soft elastic *porte-vent*, like the *trachea*, capable of changing its diameter and its length; so that all the sounds within the compass of the

double octave may readily be obtained; and, if great precautions are taken, tones even yet graver may be educed, so as to admit, in fact, no limit in this direction.

c. The glottis was generally regarded as performing the functions of a reed, until the investigations of M. Savart, who remarked that the essential principle of a reed, the periodical opening and closing of the orifice, through which the stream of air passes, is wanting in the glottis. Were the glottis a reed, the edges of the vocal ligament, which form the chink through which air passes, would require to be almost in contact, and should be alternately forced asunder by the effort of the air, and brought together by their tension. But, on the contrary, he found that the larynx of the dead subject, when left in its natural state, and gently blown into through the trachea, yielded sounds approaching to those of the voice, although the opening left between the borders of the glottis was as much as one-sixth, or even one-fourth of an inch across, and more than half an inch long.

Some physiologists have, however, retained the opinion that the vocal chords do actually come in contact and perform the office of a reed, because the voice is not destroyed by cutting the upper ligaments, while it is destroyed by cutting the lower ligaments, or the vocal chords.

The truth seems to lie between these two opinions, or rather both of them appear to be true; for, whereas the preservation of the voice after the destruction of the upper ligaments proves that the vocal chords do act in a way to produce sound of themselves, while the great change in the voice shows that an important organ has been destroyed; so the experiments of Savart render it evident that the glottis, from its very form, *must* produce a musical sound, whether the vocal chords come in contact or not; and as there is no apparent dissonance in a perfectly regulated voice from this double source of musical sound, it seems a fair conclusion that the larynx is a compound musical instrument, formed from the perfect adjustment of the musical cavity of the glottis with the delicate reed of the vocal chords. The reed thus formed is indeed of so soft a nature, that it vibrates at the least impulse, and in some

degree accommodates itself to the vibrations of the air in the glottis; and the note, thus obtained, is probably improved in strength and quality.

The voice of a man is, as every one knows, much more grave than that of a woman, and there is a corresponding difference in the dimensions of the vocal chords, which are much longer in a man than in a woman, and hence the difference in the projection of the *pomum Adami*.

d. The action of the vocal chords has been lately investigated by Mr. Willis, and elucidated by some curious experiments, from which it would appear that these *chords* might with more propriety be called *the vocal membranes.*

Mr. Willis made use of a frame of wood, $ABBBC$, fig. 171, to which was attached by its edges a thin elastic membrane, $EEEE$. Two such frames, AB, $A'B'$, were placed facing each other, as in fig. 172, (which represents a section of the apparatus employed by him,) over the mouth of an organ-pipe, $CDC'D'$, and the edges of the frames were shut in by walls, parallel to the section here given, so as not to allow of the escape of the air from the pipe.

When the frames were inclined to each other, as in fig. 172, the wind only forced the membranes apart, curving them from each other; when the frames were inclined in the opposite direction, as in fig. 173, the wind in its passage curved the membranes towards each other. But when the frames were placed, as in fig. 174, parallel to each other, or nearly so, a powerful vibration took place at the edges of the membranes, and a strong sound was produced, the pitch varying with the distance of the membranes apart.

Now a careful examination of the structure of the larynx makes it evident, that the vocal chords are placed, as in *b b*, fig. 175, with their sides nearly parallel when they vibrate in producing sound. But when the wind passes between them in silence, as in the common act of breathing, they are inclined to each other, as in fig. 170; and there is no important diminution of their tension, as supposed in the usual explanation, neither is the chink between enlarged, for they may be in actual contact without any other than a hoarse sound, like that of *clearing the throat.*

193. The *pharynx* is a large cavity above the larynx, which corresponds, in the changes of its form and of the tension of its sides, to the different notes of the voice, and is very important in reinforcing them by its resonance.

a. In this part of the air-passage, we find an exact correspondence with the flute, or pipe, in as far as it is lengthened during the grave sounds by the descending of the trachea and larynx, and shortened in the acute by their ascent.

b. But this is not all. It appears from M. Savart's experiments, that in short pipes, and in cavities, whose other dimensions bear a considerable ratio to their length, the tone yielded is rendered much graver, when the pipe or cavity is constructed of a flexible material, capable of being agitated and set in vibration by the air, than when made of more rigid materials. He constructed a cubic box-pipe with paper, stretched on slight square frames of wood, joined together at the edges, and made it speak by an embouchure at the edge. He then observed, that so long as the paper was slightly stretched, the sound yielded by the cube was nearly as acute, as it would have been, had the whole been rigid; but that when its tension was diminished by exposing it to moist vapor, or even by wetting it, the sound descended in the scale by an interval proportioned to the degree of moisture the paper had imbibed. It was thus lowered even two whole octaves, when it grew so feeble as to be no longer audible; but, repeating the experiment in the still of night, it could yet be heard, and no limit indeed then seemed set to the descent of the sound; and even when no longer audible, the vibration of the paper sides could still be made sensible by sand strewed on them, which arranged itself in nodal lines, for the most part elliptic or circular.

The relaxation then, or increase of tension, of the soft parts which form the cavity of the mouth, pharynx, and larynx, is, no doubt, a great cause of the graduation of its tones. Whoever will sing open mouthed before a looking-glass, will not fail to be struck with the extraordinary contraction of the *uvula*, (a small pendulous

substance, which seems to hang down from the roof of the mouth,) which takes place in the higher notes. It shrinks up almost into a point, and every surrounding part seems to partake of its tension, while the dropping of the lower jaw, and the effort made in every possible way to increase the dimensions and diminish the tension of the throat and fauces generally, in singing the lower notes of the scale, sufficiently prove that the note of the glottis is reinforced in this case, as in that of acuter sounds, by the resonance of the cavity above.

194. The tongue, the cavity of the fauces, the lips, teeth, and palate, with its *velum pendulum*, and the uvula, which performs the office of a valve between the throat and nostrils, as well as, perhaps, the cavity of the nostrils themselves, are all concerned in modifying the impulse given to the breath as it issues from the larynx, and producing the various consonants and vowels, according to the different capacities and shapes of their internal cavity.

195. The different *vowel* sounds are produced, either by the lengthening and shortening of the pharynx, larynx, and *trachea*, or by merely opening and closing the lips, or placing the tongue in different positions within the mouth.

a. In 1779, the Imperial Academy of Petersburgh proposed, as one of their prize questions, an inquiry into the nature of the vowel sounds *A E I O U*, and the construction of an instrument capable of artificially imitating them. The prize was awarded to M. Kratzenstein, whose principle consisted in the adaptation of a reed, in all essential respects similar to Grenié's, where the tongue passes to and fro through the slit without contact, to a set of pipes of peculiar forms, some of them very odd ones, and for whose shapes no

other reason could be given than their success on trial. This, however, was a great step. It showed the vowel quality of a sound to be something distinct from mere pitch, and susceptible of being produced at pleasure by mechanical artifice.

b. Pursuing this idea, Mr. Willis has succeeded in educing all the vowel sounds, by a mere combination of a reed on Kratzenstein's construction, with a cylindrical pipe of variable length, and investigating the laws of their production. He relates, that, having provided an apparatus, consisting of a wind-chest, or reservoir, connected with a pair of double bellows, and opening into a porte-vent, having a free reed, on Kratzenstein's or Grenié's construction, at its termination, his first object was to verify Kempelen's account of the vowels. He, therefore, adapted his reed to the bottom of a funnel-shaped circular cavity, open at top, as in fig. 176, which represents a section of the apparatus, and on making the reed speak, and placing his hand in various positions pointed out by Kempelen within the funnel, he obtained the vowels *A* (as in hard), *E* (as *A* in lame), *I* (as *E* in peep), *O* (as in hole), *U* (as in rude), very distinctly. On using, however, a shallower cavity, these positions became unnecessary, and the hand might be replaced by a flat board slided over the mouth of the cavity; and by using a very shallow funnel, as represented in fig. 177, he succeeded in obtaining the whole series in the order *U, O, E, A, I.*

c. Being thus led away from Kempelen's experiment, he proceeded to try the effect of adapting to the reed cylindrical tubes, whose length could be varied at pleasure by sliding joints. This was easily accomplished by fixing the reed with its porte-vent into the end of a pretty long horizontal pipe, coming off from the wind-chest, over which on its outside, a tube, open at both ends, was made to slide on leather, wrapped round it in the manner of a piston, and capable of being lengthened, by the attachment of pipes of similar tube of its own length to any extent. He thus describes the results so obtained. Let *a b c d*, fig. 178, represent the length of the outer or sounding pipe, projecting beyond the reed, and take *a b*, *b c*, *c d*, &c. equal to the length of a stopped

pipe in unison with the reed employed, that is equal to half the length of the sonorous wave of the reed.

If, now, the pipe be drawn out gradually, the tone of the reed, retaining its pitch, first puts on in succession the vowel quantities, *I, A, E, O, U.* As the length approaches to *a c*, the same series makes its appearance in an inverted order, as represented in the diagram; then on passing the length, *a c*, in direct order again, and so on in cycles, each cycle being merely a repetition of the foregoing, but the vowels becoming less and less distinct in each successive cycle, and the distance of any given vowel from its respective central points, *a, c,* &c. being the same in all the cycles.

If another reed be adapted to the same pipes, having a different fundamental sound, or sonorous wave, the same phenomena will be produced, only that the central points of the new cycles will now be at a distance from each other equal to the sonorous wave of the new reed, but the distances of the several vowel points from the centres of the respective cycles will be the same as before.

d. When the pitch of the reed is high, so that the length, *a c*, of its wave is less than twice the distance, *a U*, corresponding to any vowel, all the vowels beyond that distance become impossible. If, for instance, *a c* be less than 2 *a h*, but greater than 2 *a O*, the series will never extend so far as *U*; but on lengthening the pipe indefinitely the succession of vowels *I E A O A E I* will be repeated. If, in like manner, still higher notes be taken for the reed, more vowels will be cut off. This, Mr. Willis remarks, is exactly the case with the human voice; female singers being unable to pronounce *U* and *O* on the higher notes of their voices. For example, the proper length for a pipe to produce *O* is that which corresponds to the note $C^{v\prime}$, two octaves above the middle *C* of a piano-forte, and beyond this note in singing it will be found impossible to pronounce a distinct *O*.

e. Cylinders of the same length, or, more generally, cavities of any figure resounding to the same note, give the same vowel, when applied to one and the same reed.

196. The different consonants are the vowel sounds checked by the tongue, lips, or teeth, accompanied by

certain murmurs and hissing sounds in the throat and mouth.

a. Thus, *P*, *T*, *K*, are respectively produced by the closing of the lips, the meeting of the tongue and palate, and the meeting of the dorsum of the tongue and palate; while *B*, *D*, *G* (hard) only differ from them in a previous murmur produced by the passage of the air into the pharynx. *M*, *N* resemble *B* and *D* in the passage of the air through the glottis; but the air, instead of being collected in the pharynx, ascends into the cavities of the face, the velum of the palate being lifted. The sounds of *V*, *Z*, *TH* (hard), *ZH*, are the same with *F*, *S*, *TH* (soft), *SH*, preceded by the murmur in the glottis.

b. The pharynx by its power of dilatation and contraction is an important agent in articulating, or forming the consonants. This is seen in the formation of *B*, *D*, and *G* (hard), which could not otherwise be produced; and, indeed, if we grasp the throat whilst speaking, so that the fingers embrace the bag of the pharynx, we shall feel that each articulate sound is attended with an action of the pharynx, and preceding each explosive letter, we shall be sensible of a distention of the throat.

By a close attention to the act of breathing, we shall perceive that whilst the distended chest falls gradually and uniformly, the bag of the pharynx is alternately distended and compressed, in correspondence with the articulated sounds. We can easily conceive that, if each impulse of the breath in speaking arose from the action of the chest, it would be attended with great and unnecessary exertion; since in proportion to the size of the reservoir, is the force required on the sides of the reservoir to produce an impulse along the tube which gives issue. Thus, Dr. Young made a comparison of the power employed by a glass blower, in propelling the air through his tube by the force of his cheeks, and in propelling it by the force of his lungs; and, calculating the ease with which the less cavity of the mouth is compressed, in comparison with the greater cavity of the chest, he concluded that the weight of four pounds would produce an operation through the smaller cavity,

equal to seventy pounds weighing on the larger cavity. The same calculation may be applied in the case of the pharynx, and the substitution of it for the larger cavity of the chest, to the great relief of the speaker, and the most important saving of muscular exertion. If each consonant and accented syllable requires the action of the whole thorax, we should find that a man, instead of being able to deliver an oration of some hours in length, would be exhausted in a few sentences; like a person who bellows and gives pain by the violence and consequent ungracefulness of his action.

197. The *Whisper* is the effect of the passage of the air through the vocal tube, whilst the position of the glottis is such that it scarcely vibrates, and produces hardly any variety of pitch.

198. The varieties of pitch in *Whistling*, which is a sort of shrill inarticulate whisper, are chiefly produced by the mechanism of the tongue.

This may be proved by depressing the tongue, by means of a thin rod passed through the corner of the mouth, during the act of whistling. The power of intonation in this function is thereby destroyed. The motion of the tongue may be felt, and its instrumentality in producing the pitch of whistling may be ascertained, by any one who can execute a shake in this sort of sound. During the shake, there is no alteration in the aperture of the lips. For, though some variation in this respect may be observed, in running through the whole compass, particularly when the note is made by inspiration, the principal agency of the aperture consists in the production of the shrillness which characterizes this function.

199. The *Falsette* is that peculiar voice in which the higher degrees of pitch are made, after the natural voice breaks, or outruns its power. The cry, scream, yell, and all shrillness, are various modes of the falsette.

a. The striking difference in quality, between the natural and the falsette voices, has created the idea of a difference in their respective mechanisms, not only as regards the kind of sound, but likewise its pitch; but after all that has been written upon this intricate subject, it is delightful to turn from the vague hypotheses of ignorance, to the modest and sound philosophy of Dr. Rush, who, without pretending to know the cause of the falsette, contents himself with showing what *it is not;* and we cannot do better than to follow his example.

b. First, then, the falsette does not seem to be produced on the larynx, as appears from an experiment of M. Bennati, repeated after M. Deleau.

M. Deleau introduced into the pharynx, through the nostril, a hollow probe, through which was forced a current of air from a pair of bellows. As soon as he felt the air strike the sides of his throat, he held his breath, moving the organs of voice as if in the act of speaking; and a low voice was heard, distinctly uttering all the elements of articulate speech. To assure himself that this voice did not proceed from the larynx, he spoke at the same time in a loud tone, and two voices were heard, as of two different persons saying the same thing.

This experiment was repeated by M. Bennati, whose thorough knowledge of anatomy and music well fitted him for such investigations; and he remarked that the notes of the voice thus obtained were those of the falsette.

c. M. Bennati called the pure notes of the falsette the *surlaryngian,* because he supposed them to be made *above* the larynx, by the back parts of the mouth, which he observed to be very much contracted during these notes. But Dr. Rush has shown the insufficiency of this hypothesis from the facts; that, during the exercise of the falsette on the element *a* (as in *a* we,) very little alteration takes place in the positions of this part of the mouth; that the elements *n* and *m,* both of which are made by the passage of air from the glottis solely through the nose, can be precisely intonated in the falsette scale; that the falsette may be made by inspiration through

the nose with the mouth closed; and, further, that if we inhale through a tube, one end of which reaches beyond the soft palate, the falsette may be carried through its pitch, though the current of air in this case does not impress the soft parts at the back of the mouth; and the same is true of expiration, where the current passes into the tube, without passing the isthmus of the fauces. He also relates a case in which the falsette was produced by a female, destitute of the whole of the soft palate, that is, of the principal part of the apparatus upon which the explanation of Bennati depends.

d. We seem, therefore, to be driven for an explanation of the falsette to the resonance of the pharynx, which, from the experiments of Savart before referred to, must experience great changes in its note, from the increased rigidity of its sides during the exercise of the falsette; and this increase of rigidity appears equally from the observations of M. Bennati, and those of Dr. Rush; while the experiments of Mr. Willis prove that the position of the vocal chords may be such, as not to interfere with the note of the pharynx by any note of their own, and this, too, even when the note of the pharynx is not above their compass.

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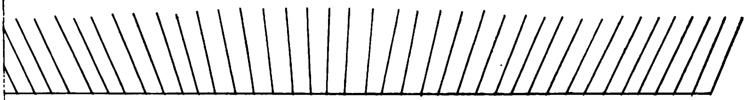

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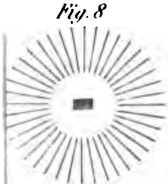

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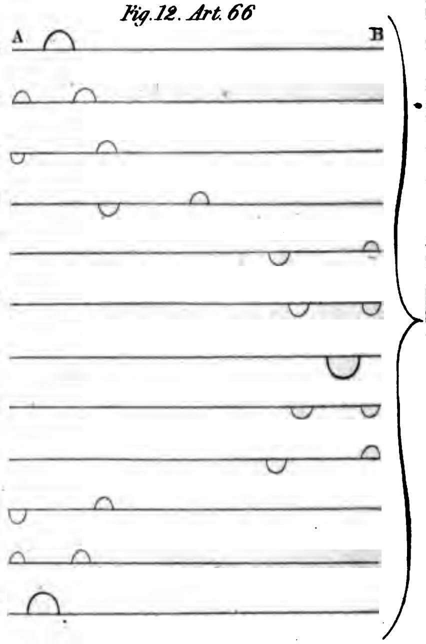

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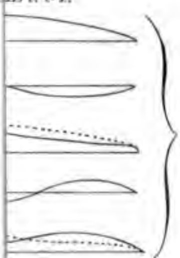

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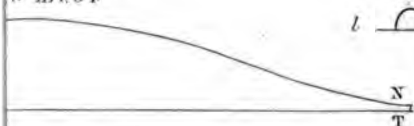

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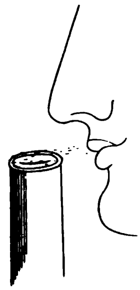

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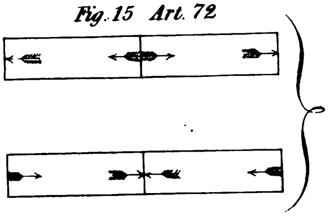

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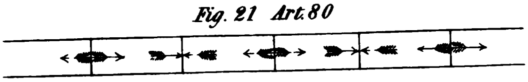

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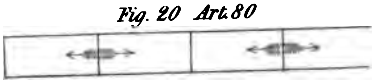

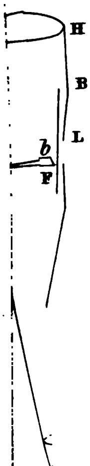

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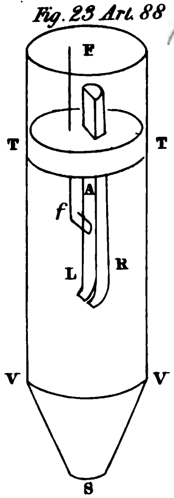

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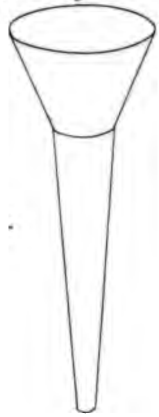

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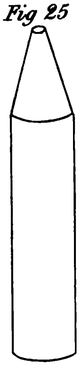

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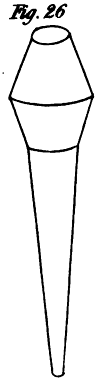

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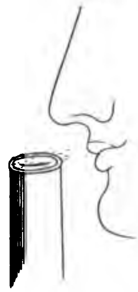

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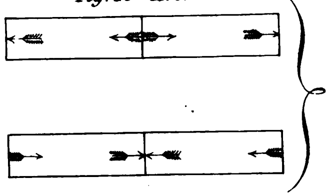

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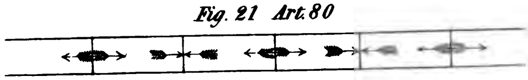

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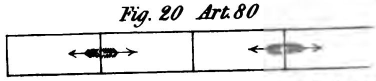

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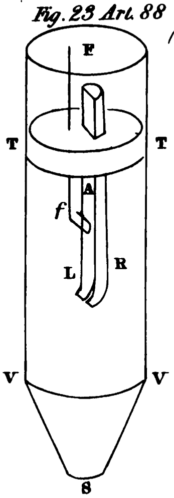

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Fig. 24

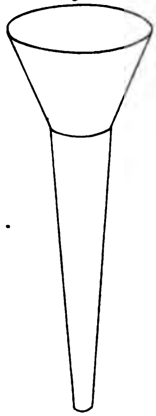


Fig 25

Fig. 26

D.

PLATE 4.

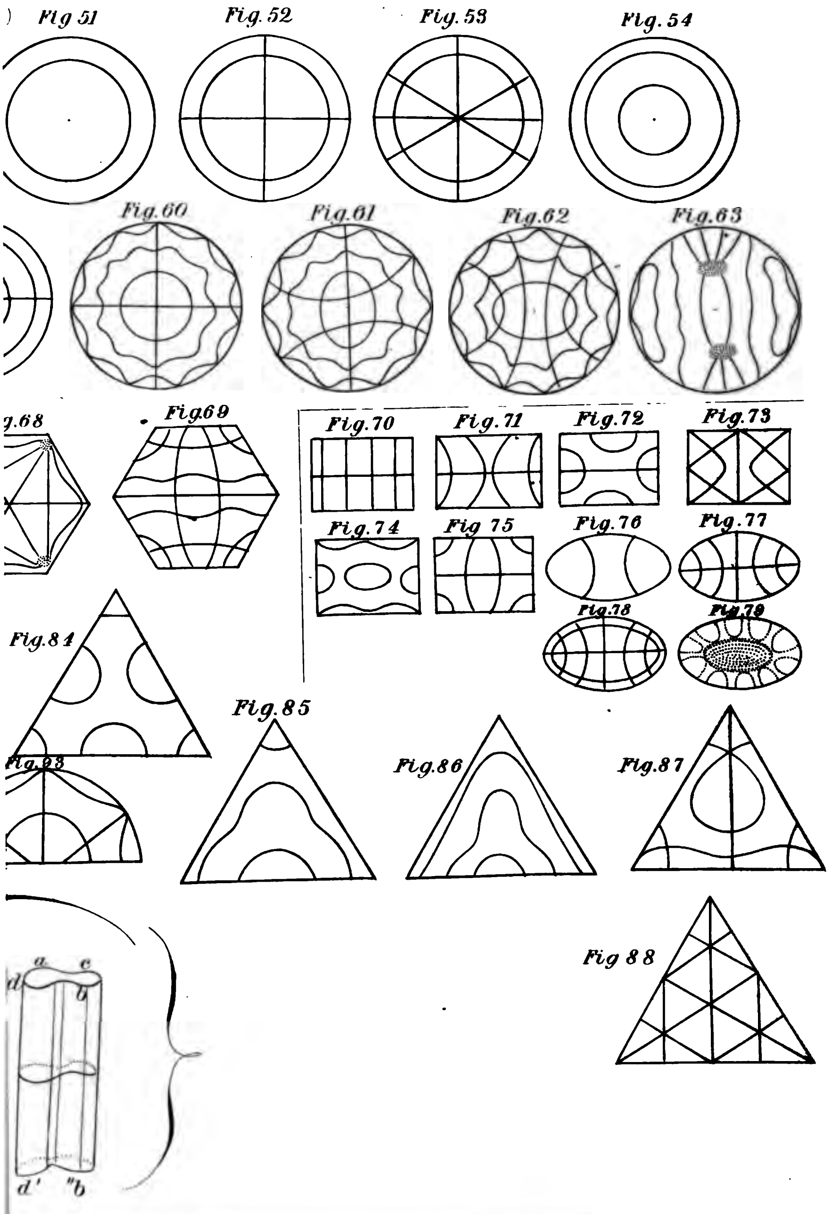


Fig 51
Fig. 52
Fig. 53
Fig. 54
Fig. 60
Fig. 61
Fig. 62
Fig. 63
g. 68
Fig. 69
Fig. 70
Fig. 71
Fig. 72
Fig. 73
Fig. 74
Fig 75
Fig. 76
Fig. 77
Fig. 78
Fig. 79
Fig. 84
Fig. 85
Fig. 86
Fig. 87
Fig. 93
Fig 88
a
c
d
b
d'
b

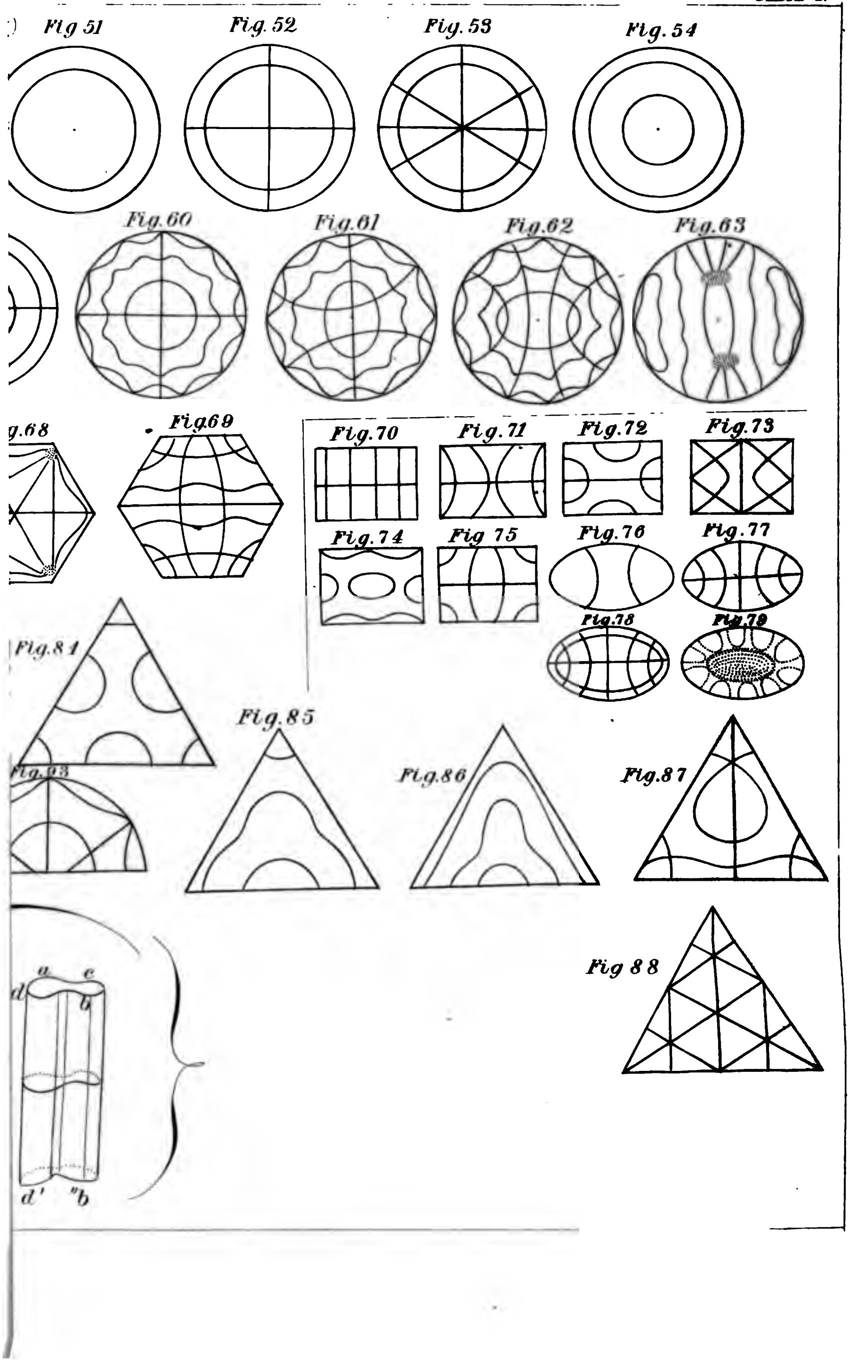
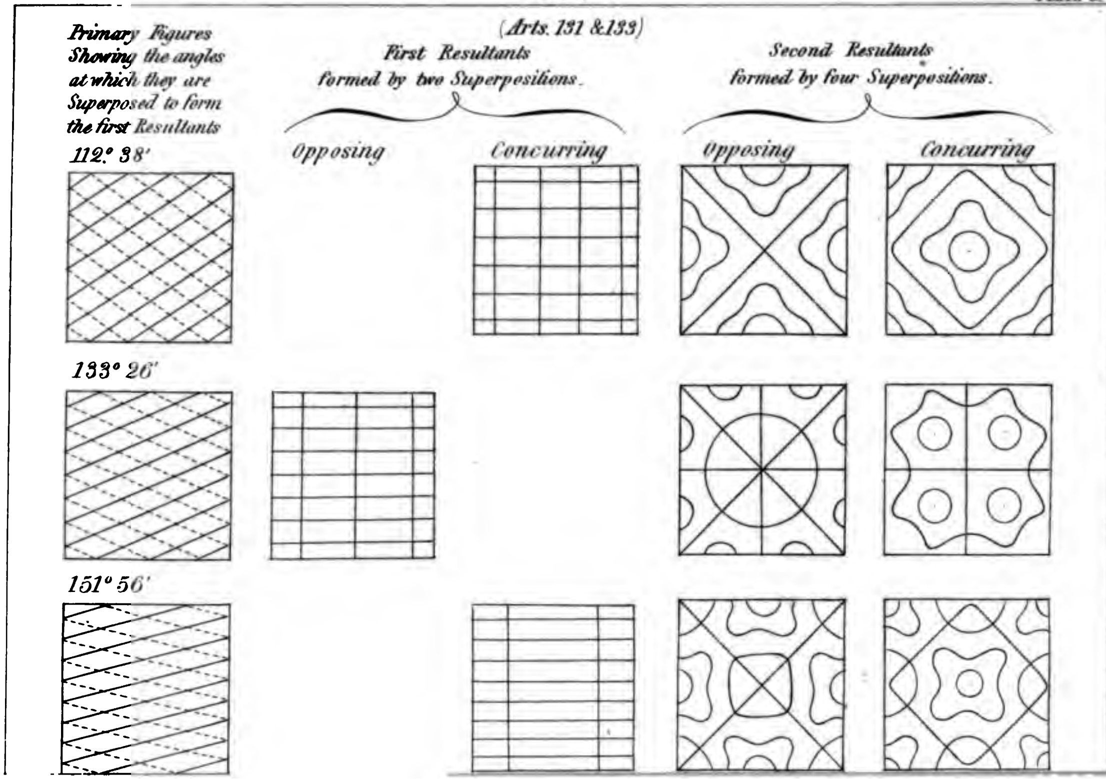
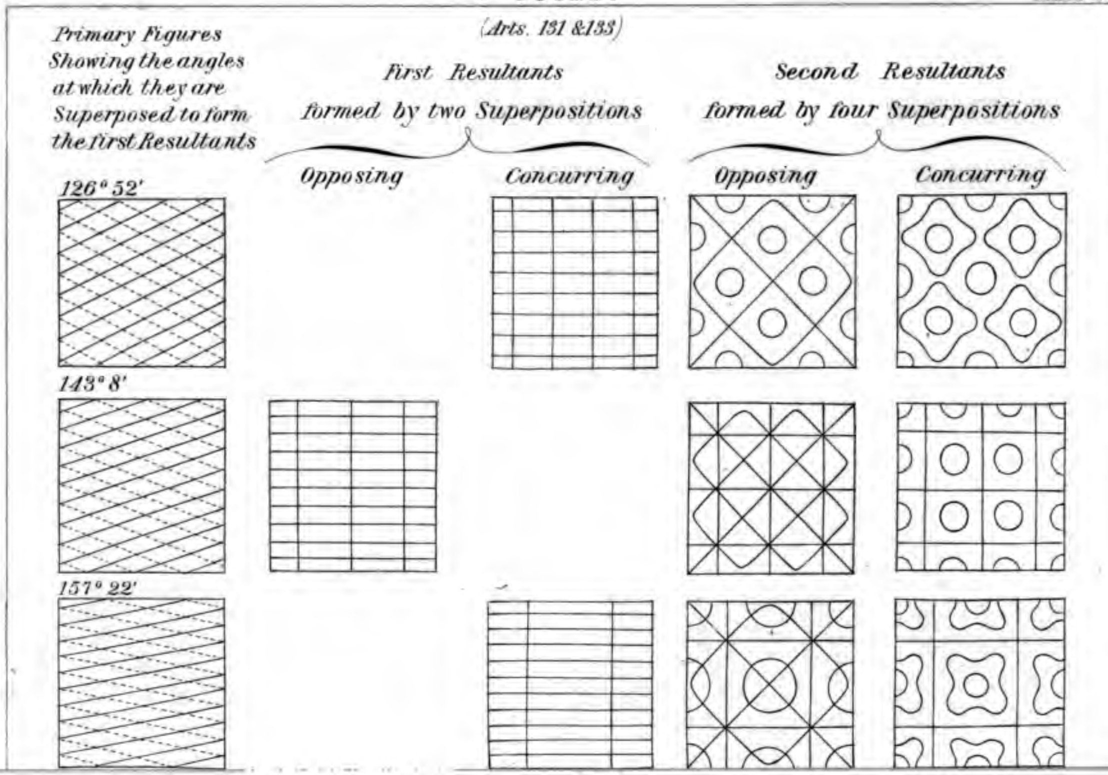


Fig 51
Fig. 52
Fig. 53
Fig. 54
Fig. 60
Fig. 61
Fig. 62
Fig. 63
g. 68
Fig. 69
Fig. 70
Fig. 71
Fig. 72
Fig. 73
Fig. 74
Fig 75
Fig. 76
Fig. 77
Fig. 78
Fig. 79
Fig. 84
Fig. 85
Fig. 86
Fig. 87
Fig 88
a
c
d
b
d'
b


(Arts. 131 & 133)
Primary Figures Showing the angles at which they are Superposed to form the first Resultants
First Resultants formed by two Superpositions.
Second Resultants formed by four Superpositions.
Opposing
Concurring
Opposing
Concurring
112° 38′
133° 26′
151° 56′


(Arts. 131 &133)
Primary Figures
Showing the angles
at which they are
Superposed to form
the first Resultants
First Resultants
formed by two Superpositions
Second Resultants
formed by four Superpositions
Opposing
Concurring
Opposing
Concurring
126° 52'
143° 8'
157° 22'

Art. 151

Fig. 118

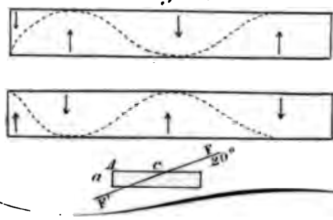
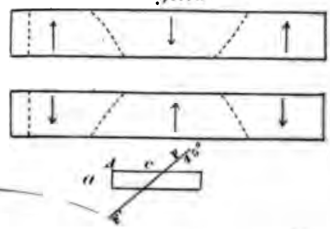


Fig. 119



Art. 161

Fig. 123

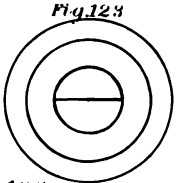


Fig. 124

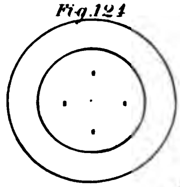
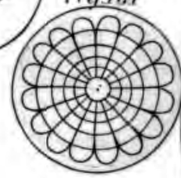


Fig. 125

Fig. 131



ig. 128

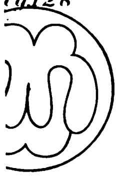


Fig. 129

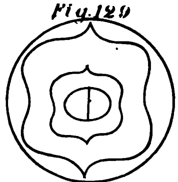


Fig. 130

Fig. 136

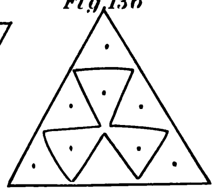


Fig. 137

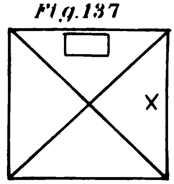


Fig. 138

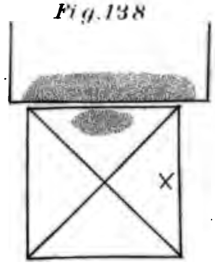


Fig. 143

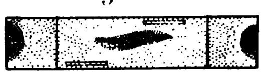
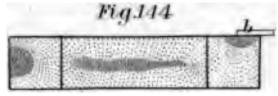


Fig. 144

a



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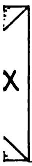


Art.172

Fig.150 Art.171

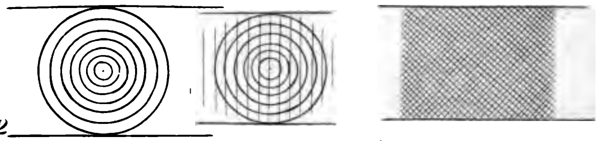


Fig.154

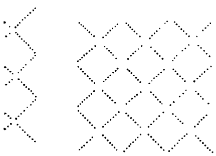
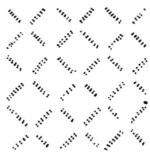


Fig.155



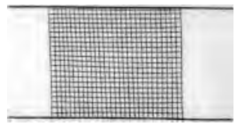
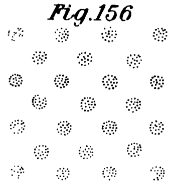


Fig.156



Art.176

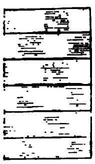


Fig.158

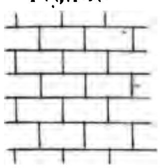
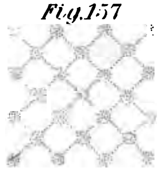


Fig.157



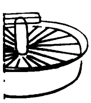


Fig.166 Art.184

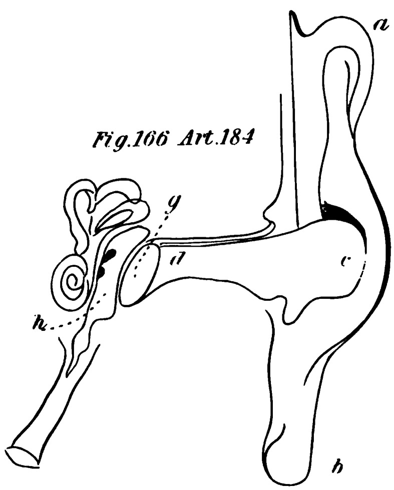


Fig.167 Art.186